Acta Cybernetica, Tom. 10, Fasc. 1-2, Szeged, 1991

Free submonoids and minimal ω-generators of R^ω

Igor Litovsky*

Abstract

Let A be an alphabet and let R be a language in A^+. An ω-generator of R^ω is a language G such that $G^\omega = R^\omega$. The language $\text{Stab}(R^\omega) = \{u \in A^* : uR^\omega \subseteq R^\omega\}$ is a submonoid of A^*. We give results concerning the ω-generators for the case when $Stab(R^\omega)$ is a free submonoid which are not available in the general case. In particular, we prove that every ω-generator of R^ω contains at least one minimal ω-generator of R^ω. Furthermore these minimal ω-generators are codes. We also characterize the ω-languages having only finite languages as minimal ω-generators. Finally, we characterize the ω- languages ω-generated by finite prefix codes.

1 Introduction

Let A be an alphabet. Given a language R in A^*, the star operation provides a language, denoted by R^*, which is the smallest submonoid of A^* containing R. Conversely with each submonoid M of A^*, we can associate the family of languages G satisfying $G^* = M$, such languages are called $*$-generators of M. To obtain the most compact possible representation of M, one can seek the smallest $*$-generator of M if any with respect to inclusion. It is well known that, if M is submonoid of A^*, then the star root of M, that is the language $(M \setminus \{\varepsilon\}) \setminus (M \setminus \{\varepsilon\})(M \setminus \{\varepsilon\}))$ is the smallest $*$-generator of M [Br].

Here we consider the ω-power operation which for each language R in A^+, gives the language R^ω of infinite words $u_1 \dots u_n \dots$ where every u_n is a word in R. Conversely, with each language R^ω, we can associate a family of languages G satisfying $G^\omega = R^\omega$. Such languages are called ω- generators of R^ω. Note that for any ω- generator G of R^ω, the language $(G^2 \setminus G)$ is an ω-generator of R^ω, too. Hence the set of ω-generators does not have a minimum, therefore we consider its minimal elements. The question about the existence of minimal ω-generators remains to be solved in the general case. Here we approach the problem in a particular case in the following way. Each word u in A^* defines a left translation on A^ω. Given an ω-language L, the language $\text{Stab}(L)$, already introduced in [St80], of words which stabilize L is a submonoid of A^*. For the case when $L = R^\omega$ and $\text{Stab}(L)$ is a

*Laboratoire Bordelais de Recherche en Informatique, Univesité de Bordeaux I, Unité associé au C. N. R. S. n° 1304. Mailing address: 351, cours de la libération, F-33405 Talence Cedex FRANCE

free submonoid, we show that $\mathrm{Stab}(R^\omega)$ is of interest for the study of minimal ω-generators of R^ω. Previously other properties of the ω-languages whose stabilizer is free have been proved in [St80]. We establish here results which, for the general case, either do not hold (we propose counter-examples) or are not yet proved. The main result (Theorem 7) states that each ω- generator of R^ω contains at least one minimal ω-generator. Furthermore these minimal ω- generators are codes. Next we are interested in the finite, if any, minimal ω-generators of R^ω. By [LaTi] such ω-languages R^ω are closed sets with respect to the usual topology on A^ω. This makes us study the minimal ω-generators of closed ω-languages. We prove that they are right-complete sets (Theorem 9). Concerning the finite minimal ω-generators of R^ω, it is proved in [LaTi] and [Li] that one can decide, given a regular language R, whether $R^\omega = F^\omega$ for some finite set F. We also characterize the properties of all minimal ω-generators being finite languages (Theorem 15) and of only one ω-generator having the smallest possible cardinality (Theorem 17). Finally we show that the case of finite prefix codes is especially easy: some finite prefix code ω-generates R^ω if and only if some finite prefix code $*$-generates the stabilizer of R^ω and R^ω is a closed ω-language (Theorem 18). Unfortunately this result cannot be generalized for a larger class of codes.

Section 2 contains definitions and notation used in the following. In Section 3 we deal with the minimal ω-generators. The finite minimal ω-generators are the topic of Sections 4 and 5. Finally the finite prefix codes as ω-generators are investigated in the last section.

2 Preliminaries

Let A be a finite alphabet. We denote by A^* and A^ω the set of all finite words, and the set of all infinite words, respectively. Infinite words are called ω-words and subsets of A^* and A^ω are called languages and ω-languages, respectively. We denote by ε the empty word and by A^+ the language $A^* \setminus \{\varepsilon\}$. The concatenation is as usual extended to A^ω.

Let X be a language in A^* and let Y be a language or an ω-language. $X^{-1}Y$ stands for the language $\{v \in A^* \cup A^\omega : xv \in Y \text{ for some } x \in X\}$. X^* stands for the smallest submonoid of A^* with respect to inclusion, containing X and we denote by $\mathrm{Root}(X^*)$ the language $(X^* \setminus \{\varepsilon\}) \setminus (X^* \setminus \{\varepsilon\})(X^* \setminus \{\varepsilon\}))$.

Let u be a word and let v be word or an ω-word. The word u is a prefix of v if and only if $v \in u(A^* \cup A^\omega)$. Given a language X, $\mathrm{Pref}(X)$ is the language $\bigcup_{x\in X} \mathrm{Pref}\ (x)$.

Let u, v be two words. The word u is a suffix of v if and only if $v \in A^*u$. Given a language X, $\mathrm{Suff}(X)$ is the language $\bigcup_{x\in X} \mathrm{Suff}(x)$.

Let C be a language in A^*. C is a code if and only if each word has at most one factorization over C. A submonoid of A^* is free if and only if its root is a code [BePe]. C is an ifl-code [St86] if and only if each ω-word has at most one ω-factorization over C that is the equality $u_1 \dots u_n \dots = v_1 \dots v_n \dots$ where $u_n, v_n \in C$, implies that $u_n = v_n$ for all $n > 0$. C is a prefix code if and only if $CA^+ \cap C = \emptyset$. Note that every prefix code is an ifl-code and every ifl-code is a code. The converses do not hold [St86].

Let P be a subset of any monoid M, P is a right-complete set in M if and only if for each u in M, there exists v in M such that uv belongs to P^* [BePe].

Let X be a language in A^*, the adherence $\mathrm{Adh}(X)$ of X ([LinSt], [BoNi]) is the ω-language $\{w \in A^\omega : \mathrm{Pref}(w) \subseteq \mathrm{Pref}(X)\}$. Recall that $\mathrm{Adh}(X)$ is a closed set with respect to the usual topology on A^ω. Moreover L is a closed ω-language if and only if $L = \mathrm{Adh}(\mathrm{Pref}\ (L))$.

Let R be a language in A^+. R^ω is the ω- power of R, that is, the ω-language $\{u_1 \dots u_n \dots : u_n \in R\}$. We denote by $[R]_\omega$ the family $\{G \subseteq A^+ : G^\omega = R^\omega\}$. $G \in [R]_\omega$ is called an ω-generator of R^ω. The ω-language R^ω is said to be finitely ω-generated [LaTi] if and only if $R^\omega = F^\omega$ for some finite language F.

The stabilizer Stab(L) of an ω-language L is the language $\{u \in A^* : uL \subseteq L\}$ [St80].

3 Minimal ω-generators in the case when stab(R^ω) is a free submonoid

This work about the minimal ω-generators of R^ω is based on the stabilizer of R^ω. Recall first the following lemma.

Lemma 1 *[St80] [LiTi] Let L be a language. Then Stab(L) is a submonoid of A^*. Furthermore, in the case when $L = R^\omega$, Stab(R^ω) contains every ω-generator of R^ω.*

Lemma 2 *Let R be a language. Then* $R^\omega = (R \setminus R(Stab(R^\omega) \setminus \{\varepsilon\}))^\omega$.

Proof. Denote $R \setminus R(Stab(R^\omega) \setminus \{\varepsilon\})$ by G. The ω-language G^ω is contained in R^ω, since G is contained in R. Moreover, we have $R \subseteq (G \cup G\text{Stab}(R^\omega))$ and thus $R^\omega \subseteq (G \cup G\text{Stab}(R^\omega))R^\omega$. Now by definition of Stab(R^ω), it follows that $R^\omega \subseteq GR^\omega$ and finally $R^\omega \subseteq G^\omega$.

□

We now state a result concerning the subsets of free submonoids.

Lemma 3 *Let M be a free submonoid in A^* and G be a subset of M. Then the language $G \setminus G(M \setminus \{\varepsilon\})$ is a code.*

Proof. Denote $G \setminus G(M \setminus \{\varepsilon\})$ by G'. Let u be a word in G'^* and assume that $u \in g_1G'^* \cap g_2G'^*$ where g_1 and $g_2 \in G'$ and g_1 is a prefix of g_2. As $G' \subseteq M$, u has only one factorization in Root(M). Thus g_2 belongs to g_1M. Since $g_2 \in G'$, g_2 is equal to g_1.

□

In view of the above lemmas, we deduce:

Proposition 4 *Let R be a language such that Stab(R^ω) is a free submonoid in A^*. For each ω-generator G of R^ω, the language $G \setminus G(\text{Stab}(R^\omega) \setminus \{\varepsilon\})$ is a code ω-generating R^ω.*

We now give a characterization of codes which uses ω-words [LiSt].

Proposition 5 *Let C be a language in A^+. C is a code if and only if for each word u in C^+, the ω-word u^ω has a single ω-factorization over C.*

Proof. Assume that C is not a code. It follows that some u in C^+ has two different factorizations over C and hence u^ω has two different ω-factorizations over C. Assume now that for some u in C^+, u^ω has two different ω-factorizations over C. That is, $u^\omega = v_1 \dots v_n \dots$ where each $v_n \in C$ and the unique factorization of u in C^+ does not start with v_1. There exist four integers i, j, k and m such that $v_1 \dots v_k = u^i u'$ and $v_1 \dots v_m = u^{i+j} u'$ where u' is a prefix of u. It follows that

$u^{i+j}u'$ has two different factorizations over $C(v_1 \ldots v_m$ and $u^j v_1 \ldots v_k)$, that is C is not a code.

□

So we can deduce a basic result for this paper.

Corollary 6 *Let C be a code in A^+. Then C is a minimal ω-generator of C^ω.*

Proof. Suppose we have a code C which is not a minimal ω-generator of C^ω. Then $(C \setminus \{v\})^\omega = C^\omega$ for some word $v \in C$. Hence $v^\omega \in (C \setminus \{v\})^\omega$ what implies that v^ω has two ω-factorizations over C. This contradicts the fact that C is a code.

□

Hence the initial question about the existence of minimal ω-generators is answered.

Theorem 7 *Let R be a language such that Stab$(^\omega)$ is a free submonoid in A^*. Eazh ω-generator G of R^ω contains at least one minimal ω-generaotr of R^ω. Furthermore, the code $G \setminus G(\text{Stab}(R^\omega) \setminus \{\varepsilon\})$ is one of these.*

Without assuming that Stab(R^ω) is free, the language $R \setminus R(\text{Stab}(R^\omega) \setminus \{\varepsilon\})$ is genrally not a minimal ω-generator of R^ω, as shown by the following example.

Example 1 *Let R be the language $\{\varepsilon, b\}\{a\}\{b\}^*$. Here Stab$(R^\omega) = R^*$. but $R \setminus R(\text{Stab}(R^\omega) \setminus \{\varepsilon\}) = R$ which is not a minimal ω-generator of R^ω, since $ab\ R^\omega$ is contained in $\{a, ab^2\}R^\omega$, which implies $(R \setminus \{ab\})^\omega = R^\omega$.*

We have actually proved that whenever Stab(R^ω) is a free submonoid, then the minimal ω-generators of R^ω are exactly the codes ω-generating R^ω. However codes can ω-generate R^ω without Stab(R^ω) being a free busmonoid, as shown below.

Example 2 *Let R be the language $\{aa, aaa, b\}$. Here Stab$(R^\omega) = R^*$ which is not a free submonoid. However the language $\{aa, aaab, b\}$ is a code ω-generating R^ω.*

4 The finite minimal ω-generators of R^ω

We have seen (Lemma 1) that Stab(R^ω) contains every ω-generator of R^ω, but it is not necessarily an ω-generator of R^ω. As a counterexample consider $R = a^*b$ where Stab$(R^\omega) = \{a, b\}^*$. However if R^ω is a closed subset of A^ω, we have the following result.

Lemma 8 *[LiTi]. Let R be a language such that R^ω is a closed subset in A^ω. Then Stab(R^ω) is the greatest ω-generator of R^ω.*

Now, in the case when R^ω is closed, we can link the notion of ω-generator of R^ω and the one of right-complete set in Stab(R^ω).

Theorem 9 *Let R and G be two langauges such that R^ω as well as G^ω are closed ω-languages. Then the following two conditions are equivalent.*

(i) G is an ω-generator of R^ω

(ii) G is a right-complete set in Stab(R^ω).

Proof. Suppose G is an ω-generator of R. Les us recall [BePe] that G is a right-complete set in a submonoid M if and only if for each word u in M, there exists v in M satisfying $uv \in G^*$. Let u be a word in Stab(R^ω), we can wtite $u^\omega = g_1 \dots g_n \dots$ where each $g_n \in G$. Hence there exist two integers k, m and a prefix u' of u such that $k < m, u^k u'$ and $u^m u'$ belong to G^+. Moreover $u^m u' = u(u^{m-k-1}(u^k u'))$, thus uv belongs to G^+ where $v = u^{m-k-1}(u^k u')$ belongs to Stab(R^ω).

Conversely, if G is a right-complete set in Stab(R^ω), $G^+ \subseteq$ Stab(R^ω) and Pref(Stab(R^ω)) $\subseteq$ Pref(G^+). Hence Pref(Stab(R^ω)) = Pref(G^+). Moreover, Pref(Stab(R^ω)) = Pref(R^ω) = Pref(R^+). Now as G^ω and R^ω are closed ω-languages, G^ω = Adh(Pref(G^ω)) and R^ω = Adh(Pref(R^ω)). It follows that $G^\omega = R^\omega$.

□

Corollary 10 *Let R be a language such that R^ω is a closed ω-language and Stab(R^ω) is a free submonoid. Let G be a language such that G^ω is a closed ω-language. Then the following conditions are equivalent.*
(i) G is a minimal ω-generator of R^ω
(ii) G is a right-complete code in Stab(R^ω).

According to [LaTi], we know that if F is a finite language, F^ω is a closed ω-language. Then as a consequence of the above result we can characterize the finite minimal ω-generators of R^ω without using the ω-power.

Corollary 11 *Let R be a language such that R^ω is a closed ω-language and Stab(R^ω) is a free submonid. Then G is a finite minimal ω-generator of R^ω if and only if G is a finite right-complete code in Stab(R^ω).*

Remark. We cannot remove the assumption of R^ω being a closed ω-language. For example, with $R = a^*b$, Stab(R^ω) is the language $\{a,b\}^*$ and $\{a,b\}$ is a right-complete code in Stab(R^ω) but it is not an ω-generator or R^ω.

In [LaTi] and [Li] characterizations are given for R^ω being finitely ω-generated. In our current case we have the following characterization which does not hold in the general case [LaTi].

Theorem 12 *Let R be a language such that R^ω is a closed ω-language and Stab(R^ω) is a free submonid. R^ω is finitely ω-generated if and only if Root(Stab(R^ω)) is a finite language.*

Proof. Assume that Root(Stab(R^ω)) is an ifinite language and that G is a finite ω- generator of R^ω. As G is right-complete in Stab(R^ω), there exists a word $g \in G$ such that the set $E = \{u \in \text{Root(Stab}(R^\omega)) : \exists v \in \text{Stab}(R^\omega) \text{ with } uv \in gG^*\}$ is infinite. Since $G \subseteq$ Stab(R^ω), $g = g_1 \dots g_k$ where each $g_i \in$ Root(Stab(R^ω)). Now since E is infinite, there exists $u_1 \in E$ such that $u_1 \neq g_1$. Then u_1 Stab(R^ω) $\cap\, g_1$ StabR^ω) $\neq \emptyset$ given a contradiction.

□

However in the case when R^ω is finitely gnerated, some ω-generators could be infinite codes, as shown below.

Example 3 *Let R be the language $\{a^2, ba, ba^2\}$. Here StabR^ω) = R^* and $\{a^2, ba\} \cup ba^2\{a^2\}^*\{ba, ba^2\}$ is an infinite code ω-generating R^ω.*

That leads to propose conditions for all minimal ω-generators of R^ω to be finite ones.

Lemma 13 *Let R be a language such that R^ω is a closed ω-language. If Root(Stab(R^ω)) is a finite ifl-code then all minimal ω-generators of R^ω are finite ifl-codes.*

Proof. Denote Root(Stab(R^ω)) by C. Assume that G is an infinite minimal ω-generator of R^ω. As C is a finite language, there exists a sequence (s_n) of C^* satisfying $s_0 = \varepsilon$ and for every integer $n, s_{n+1} = s_n r_{n+1}$ with $r_{n+1} \in C$ and $s_n C^+ \cap G$ is an infinite language. Moreover by Theorem 7, $G \cap GC^+ = \emptyset$. Hence for every integer n, s_n does not belong to G. As the ω-word $r_1 \ldots r_n \ldots$ belongs to C^ω, it is equal to $g_1 \ldots g_n \ldots$ where each $g_n \in G$. As C is an ifl-code. There exist $g \neq g'$ in G such that $gG^\omega \cap g'G^\omega \neq \emptyset$. Without loss of generality we may assume that g is a prefix of g'. Since C is an ifl-code, $g' \in gG^+$, this is a contradiciton with $G \cap GC^+ = \emptyset$.

□

The following lemma displays an important difference between regular codes and regular ifl-codes.

Lemma 14 *Let C be a regular code. If C is not an ifl-code then there exists an infinite code ω-generating C^ω.*

Proof. C being not an ifl-code, there exist words $\alpha, \beta \in C$ such that $\alpha \neq \beta$ and $\alpha C^\omega \cap \beta C^\omega \neq \emptyset$. Since C is regular, we deduce that $uv^\omega = u'v'^\omega$ for some $u \neq u'$ such that $u \in \alpha C^{i-1}, u' \in \beta C^{i-1}, v \in C^i$ and $v' \in C^i$. Moreover the language $uv^*(C^i \setminus \{v\}) \cup (C^i \setminus \{v\})$ is an infinite ω-generator of R^ω, which is a code since C^i is a code.

□

Noting that a finite language is a regular language and according to Lemmas 13 and 14, we state.

Theorem 15 *Let R be a language such that Stab(R^ω) is a free submonoid. All minimal ω-generators of R^ω are finite languages if and only if R^ω is a closed ω-language and Root(Stab(R^ω)) is a finite ifl-code.*

Remark. As shown by the following example, we cannot remove the assumption that Stab(R^ω) is a free subonoid.

Example 4 *Let R be the language $\{\varepsilon, b\}\{a, ab\}^*$. R is not a code, Stab(R^ω) $= R^*$ and Root(Stab(R^ω)) $= R$. However, by using the fact that Pref(R^+))$\cap$ Suff (R^+) $= R^* \cup \{b\}$, we can prove that all minimal ω-generators of R^ω are finite langauges.*

As a consequence of Theorem 15, we characterize the minimal ω-generators of the whole language A^ω.

Corollary 16 *Let A be a finite alphabet. A language G is a minimal ω-enerator of A^ω if and only if G is a finite maximal prefix code in A^*.*

5 Uniqueness of the ω-generator of smallest cardinality

When R^ω is finitely ω-generated, there is obviously a smallest integer that can be the cardinality of some ω-generator of R^ω. But several ω-generators can have that integer for cardinality. For example, consider $R = \{aa, aaa, b\}$ where $\{aa, aaab, b\}$ is also an ω-generator of smallest cardinality. Here we seek languages R^ω such that only one ω- generator is of smallest cardinality.

Theorem 17 *Let R be a language such that R^ω is a closed ω-language and Stab(R^ω) is a free submonoid. Then the following conditions are equivalent.*
(i) Root(Stab(R^ω)) is the single ω-generator of smallest cardinality for R^ω
(ii) $2 \leq$ Card(Root(Stab(R^ω))) $< \infty$.

Proof. Denote Root(Stab(R^ω)) by C. If Card(C) $= 1$, then of course there are infinitely many ω-generators of cardinality 1. If C is infinite, then in view of Theorem 12, R^ω is not finitely ω- generated and all ω-generators are infinite languages.

Conversely, suppose $G \neq C$ is an ω-generator of smallest cardinality for R^ω. Let $g = cu$ be a word of G factorised by $c \in C$ and $u \in C^+$ (g exits since $G \neq C$). The language $(G \setminus \{g\}) \cup \{c\}$ is an ω-generator of smallest cardinality for R^ω. Step by step we obtain an ω-generator such as $(C \setminus \{c\}) \cup \{cu\}$ where $c \in C$ and $u \in C^+$. By factorizing u in $c'u'$, we can easily verify that $(C \setminus \{c\}) \cup \{cc'\}$ is an ω-generator of R^ω. Hence $(C \setminus \{c\})C \cup \{cc'\}$ is an ω-generator of R^ω, properly contained in C^2 : a contradiction since C^2 is a code and consequently C^2 is a minimal ω-generator of R^ω.

□

6 Case of finite prefix codes

In Section 3 we have seen that the language Stab(R^ω) does not allow us to characterize the languages R^ω ω-generated by a code. However for the finite prefix codes we have the following result.

Theorem 18 *Let R be a language. Then the following conditions ae equivalent.*
(i) $R^\omega = P^\omega$ for some finite prefix code P.
(ii) R^ω is a closed ω-language and Stab(R^ω) $= P^*$ for some finite prefix code P.

Proof. If R^ω is a closed ω- language and Stab(R^ω) $= P^*$ ffor some finite prefix code P, then $R^\omega = P^\omega$.

Conversely, let P be a finite prefix code such that $P^\omega = R^\omega$.

First $(^*)^{-1}$ Stab(R^ω) = Stab(R^ω). Indeed, let $uv \in$ Stab(R^ω) where $u \in P^*$. As $uvP^* \subseteq$ Pref(P^ω), for each z in P^*, there exists y in A^* such that $uvzy \in P^*$. P being a prefix code, $(P^*)^{-1}P^* = P^*$, hence $vzy \in P^* >$, that is $v \in$ Stab(R^ω).

Secondly (Stab(R^ω))$^{-1}$ Stab(R^ω) $\subseteq$ Stab(R^ω). Indeed, assume that $z \in$ (Stab(R^ω))$^{-1}$ (Stab(R^ω)). Then Stab(R^ω)$\cap$ (Stab(R^ω))$z^{-1} \neq \emptyset$. Let u be a word in Stab(R^ω)$\cap$ (Stab(R^ω))z^{-1} such that no any suffix of u is in Stab(R^ω)$\cap$ (Stab(R^ω))z^{-1}. As $u^\omega \in P^\omega$, there exist two words u_1, u_2 in A^* such that $u = u_1u_2$ and $u^iu_1 \in P^+$ and $u^{i+j}u_1 \in P^+$. Hence u_2, which is equal to $(u^iu_1)^{-1}u^{i+1}$, belongs to Stab(R^ω) according to the first point. Ditto u_2z belongs to Stab(R^ω), hence $u_2 \in$ Stab (R^ω)$\cap$ (Stab(R^ω))z^{-1}. It follows $u_2 = u$, next $u^i \in P^+$. Moreover $u^iz \in$ Stab(R^ω), hence $z \in$ Stab$R$($^\omega$). Finally (Pref(Stab(R^ω))* = Stab(R^ω). Indeed, let $u \in$ Stab(R^ω), $u = cu'$ for some c in Pref(Stab(R^ω)). According to the second point, $u' \in$ Stab(R^ω) and step by step we obtain Stab(R^ω) $\subseteq$ (Pref(Stab(R^ω))$^+$. This finishes the proof.

□

Finite prefix codes are particular finite ifl-codes. But R^ω cvan be ω-generated by a finite ifl-code without Stab(R^ω) being a free submonoid, as shown below.

Example 5 *Let R be the language $\{\varepsilon, b\}\{a, ab^2\}^*$. R is a finite ifl-code, hence R^ω is a closed o is a closed ω-language. However $Stab(R^\omega) = \{\varepsilon, b\}\{a, ab, ab^2\}^*$ and $Root(Stab(R^\omega)) = \{\varepsilon, b\}\{a, ab, ab^2\}$ which is not a code.*

When R^ω is ω-generated by an infinite prefix code, R^ω is never a closed ω-language and Stab(R^ω) is not necessarily an infinite prefix code.

Example 6 *Let R be the language $^*b$. R is an infinite prefix code, $Stab(R^\omega) = \{a, b\}^*$ which has $\{aa, b\}$ for root.*

Acknowledgments. The author is very deeply indebted to two referees for thorough reading of the first version of the manuscirpt. Their comments have resulted in a signifiant improvement in the exposition of the results.

References

[BePe] J. Berstel and D. Perrin, *Theory of codes;* Academic Press (1985).

[BoNi] L. Boasson and M. Nivat, *Adherences of languages;* Journal of Computer and System Sciences, 20 (1980) 285-309.

[Bu] J.R. Buchi, *On decision method in restricted second-order arithmetics;* Proc. Congr. Logic, Method. and Philos. Sci. (Stanford Univ. Press, 1962) 1-11.

[Br] J.A. Brzozowski, *Roots of Star Events;* J. ACM 14 (1967)3, 466-477.

[Do] Do Long Van, *Sur les ensembles gènèrateurs minimaux des sous-monoides de A^∞;* C.R. Acad. Sc. Paris; t. 300, Sèries I, n° 13, 1985.

[Ei] S. Eilenberg, *Automata, Languages and Machines;* Vol. A (Academic Press, New York, 1974).

[La] L.H. Landweber, *Decision problems for ω-automata,* Math. Syst. Theory 3 (1969) 376-384.

[LaTi] M. Latteux and E. Timmerman, *Finitely generated ω-languages;* Information Processing Letters 23 (1986) 171-175.

[LinSt] R. Lindner and L. Staiger, *Algebraische Codierungstheorie-Theorei der sequentiellen Codierungen;* Akademie-Verlag; Berlin 1977.

[Li] I. Litovsky, Rank of rational finitely generated ω-language; Proceedings of FCT'89, Lecture Notes in Computer Science 380, 308-317.

[LiSt] I. Litovsky and L. Staiger, *A characterization of codes via the ω-power,* (submitted paper).

[LiTi] I. Litovsky and E. Timmerman, *On generators of rational ω-power languages;* Theoretical Computer Science 53 (1987) 187-200.

[Ma] R. MacNaughton, *Testing and generating infinite sequences by a finite automaton;* Information and Control 9 (1966) 521-530.

[St80] L. Staiger, *A Note on Connected ω-languages;* EIK 16 (1980) 5/6, 245-251.

[St83] L. Staiger, *Finite-state ω-languages;* Journal of Computer and System Sciences, 27 (1983) 434-448.

[St86] L. Staiger, *On infinitary finite length codes;* Theoretical Informatics and Applications, vol. 20, n° 4 (1986) 486-494.

(Received December 18, 1990)