Descriptional Complexity of Multi-Continuous Grammars

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Abstract

The present paper discusses multi-continuous grammars and their descriptional complexity with respect to the number of nonterminals. It proves that six-nonterminal multi-continuous grammars characterize the family of recursively enumerable languages. In addition, this paper formulates an open problem area closely related to this characterization.

Key Words: multi-continuous grammars; descriptional complexity; nonterminals; recursively enumerable languages.

1 Introduction

The language theory has intensively and systematically investigated the descriptional complexity of grammars (see Chapter 4 in [1] and references therein). This investigation has achieved several characterizations of the family of recursively enumerable languages by various grammars with a reduced number of nonterminals (see [4] through [6]).

The present paper discusses the descriptional complexity of multi-continuous grammars (see [3]). It proves that six-nonterminal multi-continuous grammars characterize the family of recursively enumerable languages. In its conclusion, this paper points out some open problems closely related to this characterization.

2 Definitions

This paper assumes that the reader is familiar with the formal language theory, including selective substitution grammars (see Chapter 10 in [1])).

Let Σ be an alphabet. The cardinality of Σ is denoted by $Card(\Sigma)$. Σ^* represents the free monoid generated by si under the operation of concatenation. The unit of Σ^* is denoted by ε. Set $\Sigma^+ = \Sigma^* - \Sigma^* - \{\varepsilon\}$; algebraically, Σ^+ is the free semigroup generated by Σ under the operation of concatenation. For $w \in \Sigma^*$, $|w|$ denotes the length of w and *subword*(w) is defined as *subword*$(w) = \{x : x \in V^*$ and x is a subword of $w\}$.

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The **bold** symbols have special meaning hereafter. If a is a symbol, then **a** means that the original symbol, a, is *activated.* Analogously, for an alphabet Σ,

$$\mathbf{\Sigma} = \{\mathbf{a} : a \in \Sigma\} \text{ and } \{\mathbf{x} : x \in \Sigma^+\}.$$

Define the homomorphism, ι, from $(\mathbf{\Sigma} \cup \Sigma)^*$ to Σ^* as

$$\iota(\mathbf{a}) = a \text{ and } \iota(a) = a$$

for all $a \in \Sigma$.

An *EOS system* is quadruple

$$E = (\Sigma, P, S, T),$$

where Σ is an alphabet, $T \subseteq \Sigma$, $S \in \Sigma - T$, and P is a finite substitution on $\Sigma + *$. An *EOS-based s-grammar,* G, is a quintuple

$$G = (\Sigma, P, S, T, K),$$

where Σ, P, S, and T have the same meaning as in an EOS system, and $K \subseteq (\mathbf{\Sigma} \cup \Sigma)^*$. Let $u, v \in \Sigma^*$. *G directly derives v from u,* symbolically denoted as

$$u \Rightarrow v,$$

if either $u = S$ and $v \in P(S)$ or there exists a natural number, n, so

1. $u = a_1 \dots a_n$ with $a_i \in T$ for all $i = 1, \dots, n$
2. $w = b_1 \dots b_n, w \in K$, and $\iota(w) = u$
3. $v = x_1 \dots x_n$ with $x_i \in P(a_i)$ if $b_i \in \mathbf{\Sigma}$, and $x_i = a_i$ if $b_i \in \Sigma$ for each $i = 1, \dots, n$.

Instead of $x \in P(a)$, this paper writes $a \to x$ hereafter. In the standard manner, extend $\Rightarrow$to $\Rightarrow^n$, where $n \geq 0$. Based on $\Rightarrow^n$, define $\Rightarrow^+$ and $\Rightarrow^*$. The *language of* $G, L(G)$,

is defined as

$$L(G) = \{w \in T^* : S \Rightarrow^* w\}.$$

Let m be a natural number, and let $G = (\Sigma, P, S, T, K)$ be an EOS-based *s*-grammar. G is an *m-continuous grammar* if for some $n \geq 1$,

$$K = K_1 \cup \dots \cup K_n$$

so that for $i = 1, \dots, n$,

$$K_i = \Omega_1 \mathbf{\Pi_1} \Omega_2 \dots \Omega_m \mathbf{\Pi_m} \Omega_{m+1},$$

where

$\Omega_j \in \{V^* : V \subseteq \Sigma\}$ for $j = 1, \dots, m + 1$

$\Pi_k \in \{W^+ : W \subseteq \Sigma\}$ for $k = 1, \ldots, m$.

G is a *multi-continuous grammar* if G represents an m-continuous *grammar* for some $m \geq 1$. A *queue grammar* (see [2]) is a sixtuple, $Q = (V, T, W, F, R, g)$, where V and W are alphabets satisfying $V \cap W = \emptyset, T \subseteq V, F \subseteq V, F \subseteq W, R \in (V-T)(W-F)$, and $g \subseteq (V \times (W-F)) \times (V^* \times W)$ is a finite relation such that for any $a \in V$, there exists an element $(a, b, x, c) \in g$. If there exist $u, v \in V^*W, a \in V, r, z \in V^*$, and $b, c \in W$ such that $(a, b, z, c) \in g, u = arb$, and $v = rzc$, then Q directly derives v from u, denoted by $u \Rightarrow v$. In the standard manner, define $\Rightarrow^n, \Rightarrow^+$, and $\Rightarrow^*$. A derivation of the form $R \Rightarrow^* wf$ with $w \in T^*$ and $f \in F$ is a successful derivation. The language of $QL(Q)$, is defined as $L(Q) = \{w \in T^* : R \Rightarrow^* wf \text{ where } f \in F\}$.

3 Results

The present section demonstrates that the family of recursively enumerable languages equals the family of languages g 1 by six-nonterminal multicontinuous grammars.

Lemma 1 *Let*

$$Q = (V, T, W, FR, g)$$

be a queue grammar. Then, there exists a six-nonterminal multi-continuous grammar, G, satisfying

$$L(G) - \{\varepsilon\} = L(Q) - \{\varepsilon\}.$$

Proof: Let

$$Q = (V, T, W, F, R, g)$$

be a queue grammar. Without any loss of generality, assume that

$$(V \cup W) \cap \{0, 1, 2, 3, X, Y\} = \emptyset.$$

<u>Construction:</u>

For some $n \geq 2^{\#(V \cup W)}$, introduce the following four mappings $-\beta, \rho, \sigma$, and δ:

1. Define an injection β from $(V \cup W)$ to $(\{0,1\}\{3\})^n$. In the standard manner, extend β so it is defined from $(V \cup W)^*$ to $((\{0,1\}\{3\})^n)^*$. β^{-1} represents the inverse of β.

2. Let ρ be the mapping from $(\{0,1\}\{3\})^n((\{0,1\}\{3\})^n \cup T)^*$ to $((\{0,1\}\{3\})^n \cup T)^*(\{0,1\}\{3\})^n$ defined as

   $$\rho(ax) = xa$$

   for all $a \in (\{0,1\}\{3\})^n$ and $x \in ((\{0,1\}\{3\})^n \cup T)^*$.

3. Let σ be the mapping from $(T \cup \{0,1,2,3\})^*$ to $(T \cup \{0,1,3\})^*$ defined as

   $$\sigma(a) = a \text{ for all } a \in T \cup \{0,1,3\} \text{ and } \sigma(2) = \varepsilon.$$

4. Let δ be the mapping from $\{0,1,3\}^*$ to $\{X,Y,3\}^*$ defined as

$$\delta(0)=X, \delta(1)=X \text{ and } \delta(3)=3.$$

Set

$$m=\max\{|\beta(x)| : (a,b,x,c)\in g \text{ and some } a\in W-F, c\in W, \text{ and } b\in V\}+6n+2.$$

Define the following m-continuous grammar

$$G=(T\cup\{0,1,2,3,X,Y\},P,2,T,K),$$

where

$$\begin{aligned}
P \;=\; & \{2\to\beta(b)2\beta(a)X^{m-2|\beta(b)\beta(a)|-2}2 : a\in V-T, b\in W-F, ab=R\}\\
\cup\; & \{a\to a : a\in T\cup\{0,1,2,3\}\}\\
\cup\; & \{3\to 32, 2\to\varepsilon\}\\
\cup\; & \{i\to\delta(i) : i=0,1,3\}\\
\cup\; & \{a\to\varepsilon : a\in\{X,Y,3\}\}\\
\cup\; & \{2\to X^j2 : j=1,\ldots,m-4n-2\}\\
\cup\; & \{2\to X^j : j=1,\ldots,m-2n-1\}\\
\cup\; & \{2\to\beta(c)2 : c\in W\}\\
\cup\; & \{2\to\beta(x)X^{m-|\beta(abcx)|-2}2 : x\in V^*, \text{ and } (a,b,x,c)\in g, \text{ where}\\
& \quad a,c\in W-F \text{ and } b\in V\}\\
\cup\; & \{2\to\beta(x)X^{m-|\beta(abcx)y|-2}2 : x\in V^*, y\in T^+, \text{ and } (a,b,xy,c)\in g, \text{ for some}\\
& \quad a\in W-F, c\in W, \text{ and } b\in V\}\\
\cup\; & \{2\to yX^{m-|\beta(abc)y|-2}2 : y\in T^*, \text{ and } (a,b,y,c)\in g, \text{ for some}\\
& \quad a\in W-F, c\in W, \text{ and } b\in V\}.
\end{aligned}$$

Furthermore,

$$K=K_1\cup K_2\cup K_3\cup K_4\cup K_5\cup K_6$$

where K_1 through K_6 are constructed as follows. Initially, set

$$K_i=\emptyset$$

for $i=1,\ldots,6$. Then, extend K_1 through K_6 in the following way.

A. If

$$(a,b,x,c)\in g, \text{ where } b,c\in W, a\in V, \text{ and } x\in V^*$$

then

$$\begin{aligned}
K_1 := K_1 \cup \{&\{\mathbf{b_1}\}^+\{3\}^+\ldots\{\mathbf{b_n}\}^+\{3\}^+\{2\}^+\{\mathbf{a_1}\}^+\{3\}^+\ldots\{\mathbf{a_n}\}^+\{3\}^+\\
&(\{0,1,3\}\cup T)^*\mathbf{H_1}\ldots\mathbf{H_{m-|\beta(ba)|-2}}\{2\}^+\},
\end{aligned}$$

where
$a_i, b_i \in \{0,1\}$ for $i = 1, \ldots, n$
$a_1 3 \ldots a_n 3 = \beta(a)$
$b_1 3 \ldots b_n 3 = \beta(b)$
$H_j = \{X\}^+$, for all $j = 1, \ldots, m - 4n - 2$

$$\begin{aligned} K_2 := K_2 \cup \{&\{\mathbf{b_1}\}^+\{\mathbf{3}\}^+ \ldots \{\mathbf{b_n}\}^+\{\mathbf{3}\}^+\{\mathbf{a_1}\}^+\{\mathbf{3}\}^+ \ldots \{\mathbf{a_n}\}^+\{\mathbf{3}\}^+\{\mathbf{2}\}^+ \\ &(\{0,1,3\} \cup T)^*\mathbf{H_1} \ldots \mathbf{H_{m-|\beta(ba)|-2}}\{\mathbf{2}\}^+\}, \end{aligned}$$

where
$a_i, b_i \in \{0,1\}$ for $i = 1, \ldots, n$
$a_1 3 \ldots a_n 3 = \beta(a)$
$b_1 3 \ldots b_n 3 = \beta(b)$
$H_j = \{X\}^+$, for all $j = 1, \ldots, m - 4n - 2$

$$\begin{aligned} K_3 := K_3 \cup \{&\delta\{(\mathbf{b_1})\}^+\{\mathbf{3}\}^+ \ldots \{\delta(\mathbf{b_n})\}^+\{\mathbf{3}\}^+\{\delta(\mathbf{a_1})\}^+\{\mathbf{3}\}^+ \ldots \\ &\{\delta(\mathbf{a_n})\}^+\{\mathbf{3}\}^+\{\mathbf{c_1}\}^+\{\mathbf{3}\}^+ \ldots \\ &\{\mathbf{c_n}\}^+\{\mathbf{3}\}^+\{\mathbf{2}\}^+(\{0,1,3\}^*\{\mathbf{d_1}\}^+\{\mathbf{3}\}^+ \ldots \\ &\{\mathbf{d_{|x|}}\}^+\{\mathbf{3}\}^+\mathbf{H_1} \ldots \mathbf{H_{m-|\beta(bacx)|-2}}\{\mathbf{2}\}^+\}, \end{aligned}$$

where
$a_i, b_i, c_i, d_i \in \{0,1\}$, for $i = 1, \ldots, n$
$a_1 3 \ldots a_n 3 = \beta(a)$
$b_1 3 \ldots b_n 3 = \beta(b)$
$c_1 3 \ldots c_n 3 = \beta(c)$ for some $c \in V$
$d_1 3 \ldots d_{|x|} 3 = \beta(x)$
$H_j = \{X\}^+$, for all $j = 1, \ldots, m - |\beta(bacx)| - 2$.

B. If

$$x \in V^*, y \in T^+, \text{ and } (a, b, xy, c) \in g \text{ for some } b, c \in W \text{ and } a \in V$$

then

$$\begin{aligned} K_4 := \quad K_4 \cup \{&\{\delta(\mathbf{b_1})\}^+\{\mathbf{3}\}^+ \ldots \{\delta(\mathbf{b_n})\}^+\{\mathbf{3}\}^+\{\delta(\mathbf{a_1})\}^+\{\mathbf{3}\}^+ \ldots \\ &\{\delta(\mathbf{a_n})\}^+\{\mathbf{3}\}^+\{\mathbf{c_1}\}^+\{\mathbf{3}\} \ldots \\ &\{\mathbf{c_n}\}^+\{\mathbf{3}\}^+\{\mathbf{2}\}^+\{0,1,3\}^*\{\mathbf{d_1}\}^+\{\mathbf{3}\}^+ \ldots \\ &\{\mathbf{d_{|x|}}\}^+\{\mathbf{3}\}^+\{\mathbf{e_1}\}^+ \ldots \\ &\{\mathbf{e_{|y|}}\}^+\mathbf{H_1} \ldots \ldots \mathbf{H_{m-|\beta(bacx)y|-2}}\{\mathbf{2}\}^+\}, \end{aligned}$$

where
$a_i, b_i \in \{0,1\}$, for $i = 1, \ldots, n$
$a_1 3 \ldots a_n 3 = \beta(a)$
$b_1 3 \ldots b_n 3 = \beta(b)$
$c_1 3 \ldots c_n 3 = \beta(c)$ for some $c \in V$
$d_1 3 \ldots d_{|x|} 3 = \beta(x)$
$e_1 \ldots e_{|y|} = y$
$H_j = \{X\}^+$, for all $j = 1, \ldots, m - |\beta(x)| - |y| - 6n - 2$.

C. If

$$x \in T^* \text{ and } (a,b,x,c) \in g \text{ for some } b,c \in W \text{ and } a \in V$$

then

$$K_5 := K_5 \cup \{\{\delta(\mathbf{b_1})\}^+\{\mathbf{3}\}^+ \dots \{\delta(\mathbf{b_n})\}^+\{\mathbf{3}\}^+\{\delta(\mathbf{a_1})\}^+\{\mathbf{3}\}^+ \dots \{\delta(\mathbf{a_n})\}^+\{\mathbf{3}\}^+\{\mathbf{c_1}\}^+\{\mathbf{3}\} \dots \{\mathbf{c_n}\}^+\{\mathbf{3}\}^+\{\mathbf{2}\}^+\{0,1,3\}^* \mathbf{T}^+\{\mathbf{e_1}\}^+ \dots \{\mathbf{e}^+_{|\mathbf{x}|}\mathbf{T}^*\mathbf{H_1} \dots \mathbf{H}_{\mathbf{m}-|\beta(\mathbf{bac})\mathbf{x}|-\mathbf{6n}-\mathbf{3}}\{\mathbf{2}\}^+\},$$

where
$a_i, b_i \in \{0,1\}$, for $i = 1, \dots, n$
$a_1 3 \dots a_n 3 = \beta(a)$
$b_1 3 \dots b_n 3 = \beta(b)$
$c_1 3 \dots c_n 3 = \beta(c)$ for some $c \in V$
$e_1 \dots e_{|x|} = x$
$H_j = \{X\}^+$, for all $j = 1, \dots, m - |x| - 6n - 3$

D. If

$$b \in F$$

then

$$K_6 := K_6 \cup \{\{\delta(\mathbf{b_1})\}^+\{\mathbf{3}\}^+ \dots \{\delta(\mathbf{b_n})\}^+\{\mathbf{3}\}^+\mathbf{H_1} \dots \mathbf{H}_{\mathbf{m}-\mathbf{2n}-\mathbf{1}}\mathbf{T}^+T^*\},$$

where
$b_i \in \{0,1\}$, for all $i = 1, \dots, n$
$b_1 3 \dots b_n 3 = \beta(b)$
$H_j = \{X\}^+$, for all $j = 1, \dots, m - |\beta(b)| - 1$.

Main Idea:

Observe that G derives no sentential form that contains a subword consisting of two identical nonterminals. Considering this essential property, examine the construction of G to see that every successful derivation simulates a successful derivation in Q. To give an insight into this simulation in greater detail, assume that Q makes this derivation step

$$avb \Rightarrow vxc$$

according to $(a,b,x,c) \in g$. By using selectors constructed in A, G simulates $avb \Rightarrow vxc$ by making the following three steps.

$$\begin{aligned} \beta(b)2\beta(av)X^{m-|\beta(ba)|-2}2 &\Rightarrow \beta(ba)2\beta(ba)2\beta(v)X^{m-|\beta(ba)|-2}2 \\ &\Rightarrow \delta(\beta(ba))\beta(c)2\beta(vx)X^{m-|\beta(bacx)|-2}2 \\ &\Rightarrow \beta(c)2\beta(vx)X^{m-4n-2}2. \end{aligned}$$

By analogy with these steps, G uses selectors constructed in B and C to simulate $Q's$ derivation steps that produce terminals appearing in the generated word. Finally, it uses a selector constructed in D to complete the simulation. As a result, $L(Q) = L(G)$.

Formal Proof (Sketch):
Hereafter, by

$$u \Rightarrow v \ [i]$$

in G, where $i \in \{1, \dots, 6\}$, this proof symbolically expresses that G makes $u \Rightarrow v$ by using a component from K_i. For brevity, the rest of this proof omits some details, which the reader can easily fill in. Examine K to see that in G, every successful derivation, $2 \Rightarrow^+ v$ with $v \in T^+$, has this form

$$\begin{array}{lllllllll}
2 & \Rightarrow & x_0 & & & & & & \\
 & \Rightarrow & x_{1_1} & [1] & \Rightarrow & x_{1_2} & [2] & \Rightarrow & x_{1_3} \ [3] \\
 & \Rightarrow & x_{2_1} & [1] & \dots & & & & \\
 & \dots & & & & & & & \\
 & \Rightarrow & x_{t_1} & [1] & \Rightarrow & x_{t_2} & [2] & \Rightarrow & x_{t_3} \ [3] \\
 & \Rightarrow & y_1 & [1] & \Rightarrow & y_2 & [2] & \Rightarrow & y_3 \ [4] \\
 & \Rightarrow & z_{1_1} & [1] & \Rightarrow & z_{1_2} & [2] & \Rightarrow & z_{1_3} \ [5] \\
 & \Rightarrow & z_{2_1} & [1] & \dots & & & & \\
 & \dots & & & & & & & \\
 & \Rightarrow & z_{h_1} & [1] & \Rightarrow & z_{h_2} & [2] & \Rightarrow & z_{h_3} \ [5] \\
 & \Rightarrow & r & [1] & \Rightarrow & v & [6], & &
\end{array}$$

where

(i) $x_0 = \beta(b)2\beta(a)X^{m|\beta(ba)|-2}2$ with $ab = R$

(ii) t is a non-negative integer, and for all $i = 0, \dots, t$, there exist $(a, b, v, c) \in g$ and $u \in V^*$ so that

$$\begin{aligned}
x_{i_1} &= \beta(ba)2\beta(u)X^{m-|\beta(ba)|-2}2 \\
x_{i_2} &= \delta(\beta(ba))\beta(c)2\beta(uv)X^{m-|\beta(bacv)|-2}2 \\
x_{i_3} &= \beta(c)2\beta(uv)X^{m-2|\beta(c)|-2}2
\end{aligned}$$

(iii) there exist $w \in V^*$ and $(a, b, vu, c) \in g$ where $v \in V^*$ and $u \in T^+$, so that

$$\begin{aligned}
y_1 &= \beta(ba)2\beta(w)X^{m-|\beta(ba)|-2}2 \\
y_2 &= \delta(\beta(ba))\beta(c)2\beta(wv)uX^{m-|\beta(bacv)u|-2}2 \\
y_3 &= \beta(c)2\beta(wv)uX^{m-2|\beta(c)|-2}2
\end{aligned}$$

(iv) h is a non-negative integer, and for all $i = 0, \dots, h$, there exist $u \in V^*, w \in T^+$, and $(a, b, v, c) \in g$ with $v \in T^*$ so that

$$\begin{aligned}
z_{i_1} &= \beta(ba)2\beta(u)wX^{m-|\beta(ba)|-2}2 \\
z_{i_2} &= \delta(\beta(ba))\beta(c)2\beta(u)wvX^{m-|\beta(bac)v|-2}2 \\
z_{i_3} &= \beta(c)2\beta(u)wvX^{m-2|\beta(c)|-2}2
\end{aligned}$$

(v) $r = \delta(\beta(b))vX^{m-|\beta(c)|-1}$ with $b \in F$.

Observe that there also exists the following derivation

$$\begin{aligned} R &\Rightarrow \rho(\beta^{-1}(\sigma(x_{1_3}))) \ldots \Rightarrow \rho(\beta^{-1}(\sigma(x_{h_3}))) \\ &\Rightarrow \rho(\beta^{-1}(\sigma(y_3))) \\ &\Rightarrow \rho(\beta^{-1}(\sigma(x_{1_3}))) \ldots \Rightarrow \rho(\beta^{-1}(\sigma(x_{h_3}))) \\ &\Rightarrow \rho(\beta^{-1}(\sigma(r))) \end{aligned}$$

in Q. Notice that $\rho(\beta^{-1}(\sigma(r))) = v$. Thus, if in $G, 2 \Rightarrow^* v$ with $v \in T^+$, then $v \in L(Q)$; therefore,

$$L(G) - \{\varepsilon\} \subseteq L(Q) - \{\varepsilon\}.$$

Notice that in Q, every successful derivation, $R \Rightarrow^* vf$ with $v \in T^+$ and $f \in F$, has this form

$$\begin{aligned} R &\Rightarrow^* d_1 d_2 \ldots d_n y_1 c_1 \\ &\Rightarrow d_2 \ldots d_n y_1 y_2 c_2 \\ &\ldots \\ &\Rightarrow d_n y_1 y_2 \cdots y_n c_n \\ &\Rightarrow y_1 y_2 \cdots y_n f, \end{aligned}$$

where

n is a natural number
$d_k \in V$, for $k = 1, \ldots, n$
$v = y_1 y_2 \cdots y_n$
$y_1 \neq \varepsilon$
$y_i \in T^*$, for $i = 2, \ldots, n$
$c_j \in W - F$, for $j = 1, \ldots, n$
$f \in F$.

Consider any derivation expressed in this way in Q, and demonstrate that there also exists

$$2 \Rightarrow^+ v$$

in G (a detailed version of this demonstration is left to the reader). Thus

$$L(Q) - \{\varepsilon\} \subseteq L(G) - \{\varepsilon\}.$$

As $L(G) - \{\varepsilon\} \subseteq L(Q) - \{\varepsilon\}$ and $L(Q) - \{\varepsilon\} \subseteq L(G) - \{\varepsilon\}$,

$$L(Q) - \{\varepsilon\} = L(G) - \{\varepsilon\}.$$

Because G has only the six nonterminals $0, 1, 2, 3, X$, and Y, Lemma 1 holds. □

Theorem 1 *The family of languages generated by six-nonterminal multi-continuous grammars coincides with the family of recursively enumerable languages.*

Proof: Obviously, every language generated by a six-nonterminal multi-continuous grammar represents a recursively enumerable language. The rest of this proof demonstrates that every recursively enumerable language is generated by a six-non terminal multi-continuous grammar.

Let L be a recursively enumerable language. Then, there exists a queue grammar, Q, such that $L(Q) = L$ (see Theorem 2.1 in [2]). By Lemma 1, there exists a six-nonterminal multi-continuous grammar,

$$G = (T \cup \{0,1,2,3,X,Y\}, P, 2, T, K),$$

satisfying $L(Q) - \{\varepsilon\} = L(G) - \{\varepsilon\}$. Consider the six-nonterminal multi-continuous grammar, G', defined as

$$G' = (T \cup \{0,1,2,3,X,Y\}, P \cup P', 2, T, K)$$

with

$$P' = \{2 \rightarrow \varepsilon\} \text{ if } \varepsilon \in L(Q), \text{ and } P' = \emptyset \text{ if } \varepsilon \notin L(Q).$$

Observe that $L(G) - \{\varepsilon\} = L(G') - \{\varepsilon\}$. Because $L(Q) - \{\varepsilon\} = L(G) - \{\varepsilon\}, L(Q) - \{\varepsilon\} = L(G') - \{\varepsilon\}$. Furthermore, by the definition of $P', \varepsilon \in L(Q)$ if and only if $\varepsilon \in L(G')$. Therefore,

$$L(G') = L(Q).$$

As $L(Q) = L$,

$$L = L(G').$$

Therefore, this theorem holds. □

Consider i-nonterminal multi-continuous grammars, where $i = 1, \ldots, 5$. What is their generative power?

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