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An application of truth functions in formalized diagnostics*

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To Professor Pál Erdős on his sixtieth birthday

§ 1.

In what follows, we shall prove some results concerning truth functions (in §§ 2—4) and apply them to the following problem (in §§ 5—6). There is a set S of objects and there are $n+1$ subsets $Z, X_1, X_2, \ldots, X_n$ of S. Let an object $s(\in S)$ be chosen arbitrarily. We are not able to decide immediately whether or not s belongs to Z; we may observe, however, the validity of any of the n relations $s\in X_i$ and we can infer to the truth of $s\in Z$ if all the relations $s\in X_1, s\in X_2, \ldots, s\in X_n$ are checked. We are interested in deciding, whether $s\in Z$ holds or not, in such a manner that a possibly small number of the relations $s\in X_i$ should be examined (successively, in a straightforward ordering).

§ 2.

Let $f(x_1, x_2, \ldots, x_n)$ be an n-ary truth function. The *rank* $\varrho(f)$ is the number of places where f takes the value $\uparrow$ (true); of course, f takes the value $\downarrow$ (false) at $2^n-\varrho(f)$ places. The *entropy* $\eta(f)$ is defined by

$$\eta(f) = \min\left(\varrho(f), 2^n-\varrho(f)\right).$$

We have $\eta(f)=\eta(\bar{f})\leqq 2^{n-1}$; furthermore, $\eta(f)=0$ exactly if f is constant.

Let $\mathfrak{A}$ be an elementary conjunction over the set $\{x_1, x_2, \ldots, x_n\}$. The number of variables occuring in $\mathfrak{A}$ is called the *length* $l(\mathfrak{A})$ of $\mathfrak{A}$.

Suppose that $\mathfrak{A}$ contains (precisely) the variables $x_{i_1}, x_{i_2}, \ldots, x_{i_l}$ $\left(l=l(\mathfrak{A})(\geqq 1)\right)$. We denote by $x_{j_1}, x_{j_2}, \ldots, x_{j_{n-l}}$ the elements of the set

$$\{x_1, x_2, \ldots, x_n\}-\{x_{i_1}, x_{i_2}, \ldots, x_{i_l}\}.$$

* The considerations of this paper have been contained in the lecture "On some combinatorial questions" presented on the colloquium "Infinite and finite sets" held at Keszthely, June 1973.

Let $f_{\mathfrak{A}}(x_{j_1}, x_{j_2}, \ldots, x_{j_{n-l}})$ be defined as the function resulting from f if constants are substituted for each of $x_{i_1}, x_{i_2}, \ldots, x_{i_l}$ such that $\mathfrak{A}$ takes the value † with the substitutions prescribed. It is obvious that $\varrho(f_{x_i})+\varrho(f_{\bar{x}_i})=\varrho(f)$. If $\mathfrak{A}$ and $\mathfrak{B}$ are elementary conjunctions (over $\{x_1, x_2, \ldots, x_n\}$) without any variable in common, then clearly $f_{\mathfrak{A}\&\mathfrak{B}}=(f_{\mathfrak{A}})_{\mathfrak{B}}$.

For a truth function f and a variable x_i of it, let the number $\lambda(f, x_i)$ and $\mu(f, x_i)$ be defined by

$$\lambda(f, x_i) = \min(\eta(f_{x_i}), \eta(f_{\bar{x}_i})),$$

$$\mu(f, x_i) = \max(\eta(f_{x_i}), \eta(f_{\bar{x}_i})).$$

It is evident that

$$\lambda(f, x_i)+\mu(f, x_i) = \eta(f_{x_i})+\eta(f_{\bar{x}_i})$$

and that $\lambda(f, x_i)$ is the smallest of the four ranks

$$\varrho(f_{x_i}),\quad \varrho(\bar{f}_{x_i}),\quad \varrho(f_{\bar{x}_i}),\quad \varrho(\bar{f}_{\bar{x}_i}).$$ [1]

Proposition 1. *We have*

$$\lambda(f, x_i) \leqq \frac{\eta(f)}{2}.$$

Proof.

Case 1: $\eta(f)=\varrho(f)$. Then

$$\varrho(f_{x_i})+\varrho(f_{\bar{x}_i}) = \varrho(f) \leqq 2^{n-1},$$

hence

$$\min(\varrho(f_{x_i}), \varrho(f_{\bar{x}_i})) \leqq \frac{\varrho(f)}{2} \leqq 2^{n-2}.$$

This implies the conclusion evidently.

Case 2: $\eta(f)=2^n-\varrho(f)(=\varrho(\bar{f}))$. The inference is analogous to Case 1 (with $\bar{f}$ instead of f).

We say that x_i is a variable *of type* α (or, for the sake of brevity, an α-variable) of the function f if

$$\lambda(f, x_i) \geqq \eta(f)-2^{n-2}.$$

In case

$$\lambda(f, x_i) < \eta(f)-2^{n-2},$$

we call x_i a variable *of type* β (or a β-variable). If $\eta(f)\leqq 2^{n-2}$, then each variable is of type α. [2]

[1] It seems to be advantageous to consider the numbers $\lambda(f, x_i)$ as basic quantities in the subsequent treatment (because the λ's can perhaps be produced in a more natural manner, than the entropies). Another possibility for treating the topics is if one omits the λ's and defines at once the critical variables by their property to be stated in the second sentence of Proposition 8.

[2] It is trivial from this remark that there exist functions all the variables of which are of type α. In case of $n=4$ and $f=x_1x_2x_3\vee x_1x_4\vee x_2x_4\vee x_3x_4$, we have $\eta(f)=8$, $\lambda(f, x_1)=\lambda(f, x_2)=\lambda(f, x_3)=3$ and $\lambda(f, x_4)=1$, hence every variable of f is of type β. In case of $n=3$ and $f=x_1\vee\bar{x}_2\bar{x}_3$, we have $(\eta f)=3$, $\lambda(f, x_1)=0$ and $\lambda(f, x_2)=\lambda(f, x_3)=1$, thus x_1 is a β-variable and x_2, x_3 are α-variables. We have seen that the three situations, being logically possible, may really occur.

Proposition 2. *If x_i is an α-variable of f, then*

$$\eta(f_{x_i})+\eta(f_{\bar{x}_i}) = \eta(f).$$

Proof.

Case 1: $\eta(f)=\varrho(f)$ and $\varrho(f_{x_i})\leqq\varrho(f_{\bar{x}_i})$. Then

$$2\varrho(f_{x_i}) \leqq \varrho(f_{x_i})+\varrho(f_{\bar{x}_i}) = \varrho(f) = \eta(f) \leqq 2^{n-1},$$

consequently,

$$2^{n-2} \geqq \varrho(f_{x_i}) = \eta(f_{x_i}).$$

Thus

$$\varrho(f_{\bar{x}_i}) = \varrho(f)-\varrho(f_{x_i}) \leqq \eta(f)-\lambda(f, x_i) \leqq 2^{n-2},$$

hence $\eta(f_{\bar{x}_i})=\varrho(f_{\bar{x}_i})$. By summarizing our considerations, we have

$$\eta(f_{x_i})+\eta(f_{\bar{x}_i}) = \varrho(f_{x_i})+\varrho(f_{\bar{x}_i}) = \varrho(f) = \eta(f).$$

We shall now mention the conditions of the remaining three cases; in any of them, the statement can be verified by an analogous inference.

Case 2: $\eta(f)=\varrho(f)$ and $\varrho(f_{\bar{x}_i})\leqq\varrho(f_{x_i})$.

Case 3: $\eta(f)=\varrho(\bar{f})$ and $\varrho(\bar{f}_{x_i})\leqq\varrho(\bar{f}_{\bar{x}_i})$.

Case 4: $\eta(f)=\varrho(\bar{f})$ and $\varrho(\bar{f}_{\bar{x}_i})\leqq\varrho(\bar{f}_{x_i})$.

Proposition 3. *If x_i is a β-variable of f, then*

$$\mu(f, x_i)-\lambda(f, x_i) = 2^{n-1}-\eta(f).$$

Proof. Similarly to the preceding proof, we can distinguish four cases; it suffices by the analogy that we carry out the proof only when $\eta(f)=\varrho(f)$ and $\varrho(f_{x_i})\leqq\varrho(f_{\bar{x}_i})$. The formula

$$2^{n-2} \geqq \varrho(f_{x_i}) = \eta(f_{x_i})$$

is valid as in the former proof.

Our next aim is to verify indirectly that

$$\eta(f_{\bar{x}_i}) = \varrho(\bar{f}_{\bar{x}_i}) < \varrho(f_{\bar{x}_i}).$$

Suppose the contrary, i.e. $\eta(f_{\bar{x}_i})=\varrho(f_{\bar{x}_i})$. Since x_i is of type β, we have

$$2^{n-2} < \varrho(f)-\lambda(f, x_i) = \varrho(f)-\min\left(\varrho(f_{x_i}), \varrho(f_{\bar{x}_i})\right) = \varrho(f)-\varrho(f_{x_i}),$$

hence

$$\varrho(f) > 2^{n-2}+\varrho(f_{x_i}) \geqq 2^{n-1} \geqq \eta(f),$$

this contradicts the supposition $\eta(f)=\varrho(f)$.

The proof (of the case treated in details) is completed by the deduction

$$\begin{aligned}\mu(f, x_i)-\lambda(f, x_i) &= |\eta(f_{x_i})-\eta(f_{\bar{x}_i})| = |\varrho(f_{x_i})-\varrho(\bar{f}_{\bar{x}_i})| = \\ &= \left|\left(\varrho(f_{x_i})+\varrho(f_{\bar{x}_i})\right)-\left(\varrho(f_{\bar{x}_i})+\varrho(\bar{f}_{\bar{x}_i})\right)\right| = \\ &= |\varrho(f)-2^{n-1}| = |\eta(f)-2^{n-1}| = 2^{n-1}-\eta(f).\end{aligned}$$

Proposition 4. *We have*

$$\eta(f_{x_i})+\eta(f_{\bar{x}_i}) \leqq \eta(f)$$

where equality or strict inequality holds according as x_i is an α-variable or a β-variable, respectively.

Proof. The statement was asserted in Proposition 2 for α-variables. If x_i is a β-variable, then

$$\mu(f, x_i) = 2^{n-1} - \eta(f) + \lambda(f, x_i) < \eta(f) - \lambda(f, x_i)$$

by Proposition 3 and the definition of β-variables.

The next assertion is an obvious consequence of Proposition 2:

Proposition 5. *If both x_i and x_j are α-variables of f, then*

$$\eta(f_{x_i}) + \eta(f_{\bar{x}_i}) = \eta(f_{x_j}) + \eta(f_{\bar{x}_j}).$$

Proposition 6. *Let x_i, x_j be two β-variables of f. If*

$$\lambda(f, x_i) \leqq \lambda(f, x_j),$$

then

$$\mu(f, x_i) \leqq \mu(f, x_j)$$

and

$$\eta(f_{x_i}) + \eta(f_{\bar{x}_i}) \leqq \eta(f_{x_j}) + \eta(f_{\bar{x}_j}).$$

Furthermore, the strict inequality in the hypothesis implies strict inequalities in the conclusion.

Proof. By Proposition 3, we have

$$\mu(f, x_i) = 2^{n-1} - \eta(f) + \lambda(f, x_i) \leqq 2^{n-1} - \eta(f) + \lambda(f, x_j) = \mu(f, x_j),$$

thus also

$$\eta(f_{x_i}) + \eta(f_{\bar{x}_i}) = \lambda(f, x_i) + \mu(f, x_i) \leqq \lambda(f, x_j) + \mu(f, x_j) = \eta(f_{x_j}) + \eta(f_{\bar{x}_j}).$$

It is clear that all of these deductions remain valid with $<$ (instead of $\leqq$) if $\lambda(f, x_i) < \lambda(f, x_j)$ is supposed.

Proposition 7. *Let x_i be an α-variable and x_j be a β-variable of f. Then*

$$\lambda(f, x_i) > \lambda(f, x_j)$$

and

$$\eta(f_{x_i}) + \eta(f_{\bar{x}_i}) > \eta(f_{x_j}) + \eta(f_{\bar{x}_j}).$$

Proof. The first inequality follows at once by comparing the definition of α-variables to that of β-variables; the second one is implied by Proposition 4.

§ 3.

We define the *critical variables* of a truth function f by the subsequent two rules (I), (II):

(I) If every variable of f is of type α, then all the variables are critical.

(II) Suppose that f has at least one β-variable. We call a variable x_i critical exactly when

$$\lambda(f, x_i) \leqq \lambda(f, x_j)$$

for each variable x_j of f.

Proposition 8. *Any n-ary function* ($n \geqq 1$) *has at least one critical variable. Let* x_i *be a critical variable, we have*

$$\eta(f_{x_i})+\eta(f_{\bar{x}_i}) \leqq \eta(f_{x_j})+\eta(f_{\bar{x}_j})$$

for an arbitrary variable x_j *of* f*; furthermore, equality holds in this formula precisely if* x_j *is also critical. If* f *has at least one* β*-variable, then all the critical variables are of type* β.

Proof. If f has α-variables only, then our statements are valid by Proposition 5.

Assume that there exists a β-variable of f. Let x_i be a critical variable. Proposition 7 implies that x_i is of type β.

Consider an arbitrary other variable x_j. If $\lambda(f, x_i) \doteq \lambda(f, x_j)$, then x_j is critical, it is of type β and Proposition 6 guarantees

$$\eta(f_{x_i})+\eta(f_{\bar{x}_i}) = \eta(f_{x_j})+\eta(f_{\bar{x}_j}).$$

If $\lambda(f, x_i) < \lambda(f, x_j)$, then

$$\eta(f_{x_i})+\eta(f_{\bar{x}_i}) < \eta(f_{x_j})+\eta(f_{\bar{x}_j})$$

follows from Proposition 7 or Proposition 6 (according as x_j is an α-variable or a β-variable).

§ 4.

In this section, we shall give a method for determining the rank of a truth function f supposing that f is given in some disjunctive normal form. It is required that the reader is familiar with the "principle of inclusion and exclusion". [3]

If $\mathfrak{A}$ is an elementary conjunction over the set $\{x_1, x_2, \ldots, x_n\}$ (considered as an n-ary function), then obviously $\varrho(\mathfrak{A}) = 2^{n-l(\mathfrak{A})}$.

Let $\mathfrak{A}_1, \mathfrak{A}_2, \ldots, \mathfrak{A}_j$ be elementary conjunctions ($j \geqq 1$). Suppose that there exists no variable x_i such that x_i occurs in non-negated form in some $\mathfrak{A}_h$ and negated in an $\mathfrak{A}_{h'}$ (where $1 \leqq h \leqq j$ and $1 \leqq h' \leqq j$). [4] Let $l(\mathfrak{A}_1 \,\&\, \mathfrak{A}_2 \,\&\, \ldots \,\&\, \mathfrak{A}_j)$ be defined as the number of *distinct* variables occurring in $\mathfrak{A}_1 \,\&\, \mathfrak{A}_2 \,\&\, \ldots \,\&\, \mathfrak{A}_j$ (i.e. as $l(\mathfrak{B})$ where $\mathfrak{B}$ is the elementary conjunction resulted by the reduction of $\mathfrak{A}_1 \,\&\, \mathfrak{A}_2 \,\&\, \ldots \,\&\, \mathfrak{A}_j$). Since $\mathfrak{A}_1 \,\&\, \mathfrak{A}_2 \,\&\, \ldots \,\&\, \mathfrak{A}_j$ is $\uparrow$ exactly when each of $\mathfrak{A}_1, \mathfrak{A}_2, \ldots, \mathfrak{A}_j$ is $\uparrow$, we have

$$\varrho(\mathfrak{A}_1 \,\&\, \mathfrak{A}_2 \,\&\, \ldots \,\&\, \mathfrak{A}_j) = 2^{n-l(\mathfrak{A}_1 \& \mathfrak{A}_2 \& \ldots \& \mathfrak{A}_j)}$$

whenever $l(\mathfrak{A}_1 \& \mathfrak{A}_2 \& \ldots \& \mathfrak{A}_n)$ is defined. [5]

Proposition 9. *If* $\mathfrak{A}_1 \vee \mathfrak{A}_2 \vee \ldots \vee \mathfrak{A}_k$ *is a disjunctive normal form representing the function* $f(x_1, x_2, \ldots, x_n)$*, then we have*

$$\varrho(f) = \Sigma 2^{n-l(\mathfrak{A}_i)} - \Sigma 2^{n-l(\mathfrak{A}_{i_1} \& \mathfrak{A}_{i_2})} + \Sigma 2^{n-l(\mathfrak{A}_{i_1} \& \mathfrak{A}_{i_2} \& \mathfrak{A}_{i_3})} -$$
$$- \ldots + (-1)^{j-1} \Sigma 2^{n-l(\mathfrak{A}_{i_1} \& \mathfrak{A}_{i_2} \& \ldots \& \mathfrak{A}_{i_j})} + \ldots$$
$$\ldots + (-1)^{k-1} \Sigma 2^{n-l(\mathfrak{A}_1 \& \mathfrak{A}_2 \& \ldots \& \mathfrak{A}_k)},$$

[3] See [3] (p. 282) or [4] (Chapter 3) or [2] (§ 22).

[4] If this supposition is not fulfilled, then we not define $l(\mathfrak{A}_1 \& \mathfrak{A}_2 \& \ldots \& \mathfrak{A}_j)$.

[5] If it is undefined, then $\varrho(\mathfrak{A}_1 \& \mathfrak{A}_2 \& \ldots \& \mathfrak{A}_j) = 0$.

where the j th summation is extended to all such j-tuples $(i_1, i_2, \dots, i_j)$ *for which* $1 \leqq i_1 < < i_2 < \dots < i_j \leqq k$ *and* $l(\mathfrak{A}_{i_1} \& \mathfrak{A}_{i_2} \& \dots \& \mathfrak{A}_{i_j})$ *is defined.*

Proof. Let the principle of inclusion and exclusion be applied under such circumstantes that the basic set H is the definition domain of f and, for each $i (1 \leqq i \leqq k)$, H_i is the set of places at which $\mathfrak{A}_i$ takes the value $\uparrow$.

§ 5.

Now we return to our original problem (exposed in § 1). We introduce some notations. For any i, let X_i^* be the difference set $S - X_i$ $(1 \leqq i \leqq n)$. Any set

$$Y = Y_1 \cap Y_2 \cap \dots \cap Y_n$$

is called an *atom*, where Y_i is either X_i or X_i^*. There exist 2^n atoms (some of them may be empty), any object $s (\in S)$ belongs to exactly one atom.

Postulate. If Y is an arbitrary atom, then either $Y \subseteqq Z$ or $Y \cap Z = \emptyset$.

Next we define the *characteristic* (truth) *function* of the system $\{Z, X_1, X_2, \dots \dots, X_n\}$. Let a full elementary conjunction $\mathfrak{A}$ over $\{x_1, x_2, \dots, x_n\}$ be given. We assign to $\mathfrak{A}$ the atom $\sigma(\mathfrak{A})$ determined in such a way that $Y_i = X_i$ or $Y_i = X_i^*$ according as x_i occurs in $\mathfrak{A}$ without or with negation $(1 \leqq i \leqq n)$. The function value is defined by what follows:

$$f(\mathfrak{A}) = \begin{cases} \uparrow & \text{if} \quad \sigma(\mathfrak{A}) \subseteqq Z \\ \downarrow & \text{if} \quad \sigma(\mathfrak{A}) \cap Z = \emptyset. \end{cases}$$

(When $\sigma(\mathfrak{A})$ is void, then $f(\mathfrak{A})$ is defined arbitrarily. The postulate guarantees that $f(\mathfrak{A})$ is defined at each place $\mathfrak{A}$.)

Algorithm. Step 1. (a) We consider the characteristic function f of the set system $\{Z, X_1, X_2, \dots, X_n\}$, we form $\eta(f)$ and the minimum of the n values $\lambda(f, x_i)$ (by comparing the $4n$ numbers $\varrho(f_{x_i})$, $\varrho(f_{\bar{x}_i})$, $\varrho(\bar{f}_{x_i})$, $\varrho(\bar{f}_{\bar{x}_i})$, by using Proposition 9).

(b) If this minimum reaches $\eta(f) - 2^{n-2}$, then we choose an arbitrary variable x_i of f. If the minimum is smaller than $\eta(f) - 2^{n-2}$, then we choose such a variable x_i which yields the minimal value of $\lambda(f, x_i)$.

(c) We check whether or not s is contained in X_i. If $s \in X_i$, then we shall perform Step 2 with f_{x_i}. If $s \notin X_i$, then Step 2 will be executed with $f_{\bar{x}_i}$.

...

Step $m (\geqq 2)$. (a) We have produced an $(n-m+1)$-ary function $f_{\mathfrak{A}}$ in Step $m-1$. If $f_{\mathfrak{A}}$ is constantly $\uparrow$, then $s \in Z$ and the algorithm is finished. If $f_{\mathfrak{A}}$ is constantly $\downarrow$, then $s \notin Z$ and the algorithm is also finished. If $f_{\mathfrak{A}}$ is non-constant, then we consider $\eta(f_{\mathfrak{A}})$ and the minimum of the $n-m+1$ values $\lambda(f, x_{j_i})$ (analogously to the part (a) of Step 1).

(b) If this minimum reaches $\eta(\mathfrak{A}) - 2^{n-m-1}$, then we choose an arbitrary variable x_{j_i} of $f_{\mathfrak{A}}$. If the minimum is smaller than $\eta(f_{\mathfrak{A}}) - 2^{n-m-1}$, then we choose such a variable x_{j_i} which yields the minimal value of $\lambda(f_{\mathfrak{A}}, x_{j_i})$.

(c) We check whether or not s is contained in X_{j_i}. If $s \in X_{j_i}$, then Step $m+1$ will be performed with $f_{\mathfrak{A} \& x_{j_i}}$. If $s \notin X_{i_j}$, then we shall execute Step $m+1$ with $f_{\mathfrak{A} \& \bar{x}_{j_i}}$.

§ 6.

This section is devoted to justifying the algorithm. We shall deal with our basic problem (see § 1 and § 5) under such circumstances that the postulate (in § 5) is valid and we know the characteristic function $f(x_1, x_2, ..., x_n)$ but we have no further information (e.g. it is unknown how the elements of S are distributed into the atoms) at beginning the procedure.

It is evident that the algorithm is completed after at most n steps.

The entropy $\eta(f)$ can be viewed as a measure of the uncertainty whether f takes one or other truth value at a randomly chosen place of its domain. Hence we consider $\eta(f)$ as the measure of uncertainty of whether $s \in Z$ or $s \notin Z$ is fulfilled.

We try to proceed towards smaller entropies, as far as possible, by checking the validity of appropriate relations $s \in X_i$ successively. In order to do this, it seems (by Propositions 4, 8) the best strategy to obtain the minimal $\eta(f_{\mathfrak{A}\&x_i})+\eta(f_{\mathfrak{A}\&\bar{x}_i})$ in each step, i.e. to continue the process with a *critical* variable of the function $f_{\mathfrak{A}}$ (where $\mathfrak{A}$ characterizes the informations being at our disposal after the earlier steps), with respect to that the formulae $s \in X_i$ and $s \notin X_i$ are assumed equiprobable.

§ 7.

The investigations described in the previous parts of the paper seem to admit some generalizations. In this final section, I mention four possibilities of generalizing them (which can be combined with each other). The subsequent list was compiled together with Dr. Gy. Pollák.

(1) More than one membership relations $s \in Z_1, s \in Z_2, ..., s \in Z_w$ should be determined simultaneously (i.e. by the same sequence of observations of whether or not $s \in X_i$).

(2) For any atom Y, we know only the probability $P(s \in Z)$ of that $s (\in Y)$ belongs to Z (possibly lying between 0 and 1), consequently, f is a stochastic truth function (in sense of [1]). We try to achieve that

$$|2P(s \in Z)-1|$$

should be significant (i.e. larger than a given number $1-\varepsilon$).

(3) For any atom Y, we know the probability of the event that $s (\in S)$ is contained in Y (this probability may differ from $1/2^n$). (The precise goal is also to be determined.)

(4) There is assigned a number (called weight) to each X_i (interpreted as the difficulty of checking of whether or not $s \in X_i$), our aim is to minimize the sum of weights of the observations performed (instead of minimizing the number of observations).

Одно применение функций алгебры логики
в формализованной диагностике

Пусть даны подмножества $Z, X_1, X_2, ..., X_n$ некоторого множества S объектов так, что каждый атом

$$Y = Y_1 \cap Y_2 \cap ... \cap Y_n.$$

(где Y_i обозначает либо X_i либо $S-X_i$) удовлетворяет одну из формул $Y \subseteq Z$ и $Y \cap Z = \emptyset$. Предположим, что для произвольного элемента $s(\in S)$ мы можем наблюдать справедливость отношений принадлежности

$$s \in X_1, \quad s \in X_2, \ldots, s \in X_n$$

в зависимом от нас порядке.

Мы интересуемся, что принадлежность $s \in Z$ имеет ли место (где s — произвольно фиксированный элемент множества S). В случае, когда известно, какие атомы являются подмножествами множества Z и какие атомы не пересекают Z (но мы не имеем никакую информацию относительно элемента s специфически), даётся стратегия для целесообразного порядка исполнения наблюдений $s \in X_i$, с целью проверки или опровержения принадлежности $s \in Z$ после (по возможности) меньше чем n наблюдений типа $s \in X_i$.

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