Certain operations with the sets of discrete states

By M. A. GAVRILOV

In memory of László Kalmár

Discrete devices are nowadays widely used in various fields. Since the contemporary discrete devices are very complex, multipurpose and high-dimensional, considerable changes in conventional design techniques which rest upon the so-called "finite automaton" model [1] are necessary.

The basic disadvantage of the existing techniques for the description of control discrete devices, viz., flow tables (for sequential machines) and state tables (for combinational automata) is that each input, internal and output state should be dealt with separately, which limits significantly the dimensionality of the problems.

A way to increase the dimensionality is to use functions which are characteristic of sets of states with some special properties such as having the same distance between states, the same value of certain variables, etc.

Some operations with characteristic functions of the sets of states are described below. Development of these operations was necessary for the design of computer-aided logical design of discrete devices.

1. Proximity of functions

Let two Boolean functions F_i and F_j be given as their sets of permit (one meaning) M^1 and forbid (zero meaning) M^0* states $M_i=M_i^1\cap M_i^0$; $M_j=M_j^1\cap M_j^0$ characterized by the functions F_i^1, F_i^0, F_j^1, F_j^0.**

Let us distinguish the following sets of states: M_{ij}^{s1}, the subset of permit states identical for both M_i and M_j; M_{ij}^{s0}, the subset of forbid states identical for both

* A permit (forbid) state is the state in which the function is equal to one (zero). Besides, there are "don't care" states ($M^{\sim}$) which are indifferent to the value of the function (it may equal either 1 or 0).

Sets of states: M^1, M^0 and $M^{\sim}$ are nonintersect in pairs and $M^1\cup M^0\cup M^{\sim}$ is equal to the set of all states, i.e., its power is 2^n, where n is the number of varibles of the functions F_i and F_j.

** Statement "function $F(A)$ characterized sets of states M^*" means that:

$$F(A)=\begin{cases}1 & \text{if } A\in M^*\\ 0 & \text{if } A\notin M^*\end{cases}$$

M_i and M_j; $M_i^{t_1}$, the subset of permit states only for M_i not contained in M_j; $M_i^{t_0}$, the subset of forbid states only for M_i not contained in M_j, $M_j^{t_1}$, the subset of permit states only for M_j not contained in M_i; $M_j^{t_0}$, the subset of forbid states only for M_j not contained in M_i; $M_i^{r_1}$, the subset of permit states in M_i contained

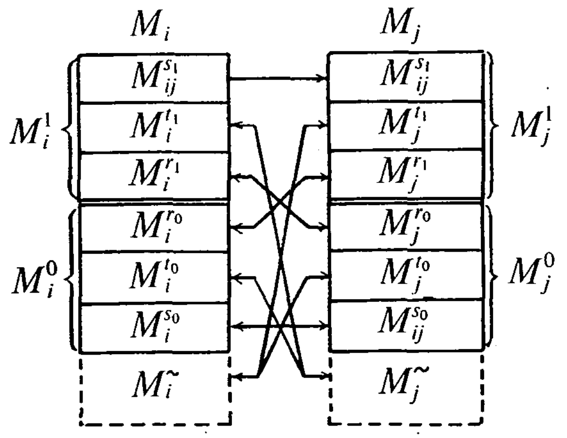

Fig. 1

in the forbid states set in M_j $(M_i^{r_1}=M_j^{r_0})$; $M_i^{r_0}$, the subset of forbid states in M_i contained in the permit states set in M_j $(M_i^{r_0}=M_j^{r_1})$ that is (fig. 1):

$$M_{ij}^{s_1} = M_i^1 \cap M_j^1;\quad M_{ij}^{s_0} = M_i^0 \cap M_j^0;$$

$$M_i^{t_1} = M_i^1 \cap M_j^{\sim};\quad M_i^{t_0} = M_i^0 \cap M_j^{\sim};\quad M_j^{t_1} = M_j^1 \cap M_i^{\sim};\quad M_j^{t_0} = M_j^0 \cap M_i^{\sim};$$

$$M_i^{r_1} = M_j^{r_0} = M_i^1 \cap M_j^0;\quad M_i^{r_0} = M_j^{r_1} = M_i^0 \cap M_j^1.$$

If the functions F_i and F_j are given by the sets of their permit and forbid states then the sets of states of classes: s, t and r are characterized by the functions:

$$\begin{aligned} F_{ij}^{s_1} &= F_i^1 F_j^1;\quad F_{ij}^{s_0} = F_i^0 F_j^0; \\ F_i^{t_1} &= F_i^1 F_j^{\sim} = F_i^1 \bar{F}_j^1 \bar{F}_j^0;\quad F_i^{t_0} = F_i^0 F_j^{\sim} = F_i^0 \bar{F}_j^1 \bar{F}_j^0; \\ F_j^{t_1} &= F_j^1 F_i^{\sim} = F_j^1 \bar{F}_i^1 \bar{F}_i^0;\quad F_j^{t_0} = F_j^0 F_i^{\sim} = F_j^0 \bar{F}_i^1 \bar{F}_i^0; \\ F_i^{r_1} &= F_j^{r_0} = F_i^1 F_j^0;\quad F_i^{r_0} = F_j^{r_1} = F_i^0 F_j^1. \end{aligned} \tag{1}$$

Let us present the sets of states F^1 and F^0 as the join of the above subsets of states. The function will then be represented as (fig. 1):

$$\begin{aligned} M_i &= [M_i^1, M_i^0] = [(M_{ij}^{s_1} \cup M_i^{r_1} \cup M_i^{t_1}),\ (M_{ij}^{s_0} \cup M_i^{r_0} \cup M_i^{t_0})] \\ M_j &= [M_j^1, M_j^0] = [(M_{ij}^{s_1} \cup M_j^{r_1} \cup M_j^{t_1}),\ (M_{ij}^{s_0} \cup M_j^{r_0} \cup M_i^{t_0})]. \end{aligned} \tag{2}$$

The proximity of the functions is measured as the power of the subsets of states M^r. If $M_i^{r_1}, M_i^{r_0}, M_j^{r_1}, M_j^{r_0}$, are empty $M_{ij}^{s_1} = M_{ij}^{s_0}$ are empty the functions F_i and F_j after introduction of the additional don't care states may be realized by the same structure (fig. 2) but in the second case the output of one of the functions was taken from an additional invertor (fig. 3). Assume the proximity of

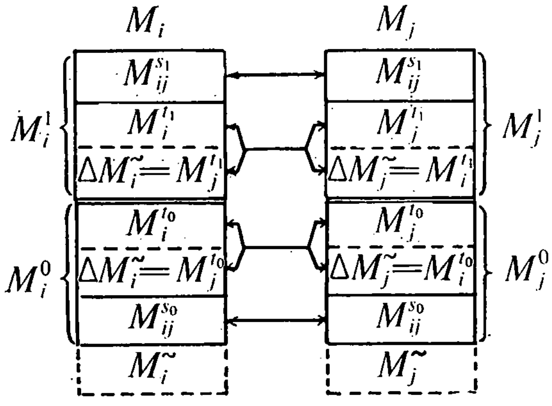

Fig. 2

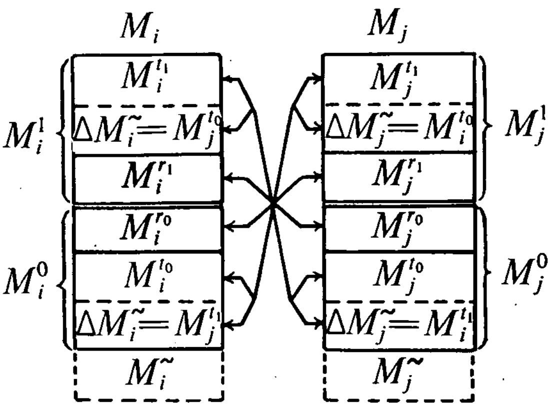

Fig. 3

the functions F_i and F_j is absolute (the distance is zero) with the corresponding functions completely connected in the first case, and maximal, with the corresponding functions inverse-completely connected in the second case.

The concept of the proximity of functions made use of in defining optimal or near-optimal architecture of realizing functions in multioutput structures. The design technique for such a realization builds the so-called "connectivity nodes" of the structure, viz., a set of functions "completely" or "inverse-completely" connected.

For functions which do not enter the connectivity nodes the distance to one of these nodes is to be found and the question answered whether the realization of these functions is connected with a connective node or a separate one.

To define these structures the operations of union intersection and complementation of subsets of states are used. If one has two sets of states M_i and M_j written in the form (2), one may write for the operations of union, intersection and complementation:

$$M_i \cup M_j = [(M_i^1 \cup M_j^1),\ (M_i^0 \cap M_j^0)] = [((M_{ij}^{s_1} \cup M_i^{t_1} \cup M_i^{r_1}) \cup$$
$$\cup (M_{ij}^{s_1} \cup M_j^{t_1} \cup M_j^{r_1})),\ ((M_{ij}^{s_0} \cup M_i^{t_0} \cup M_i^{r_0}) \cap (M_{ij}^{s_0} \cup M_j^{t_0} \cup M_j^{r_0}))].$$

The intersection of subsets of states $M_i^{r_0}$ and $M_j^{r_0}$ is empty subsets $M_i^{t_0}$ contains in subsets $M_j^{\sim}$ and subsets $M_j^{t_0}$ contains in subsets $M_i^{\sim}$. Therefore we shall have:

$$M_i \cup M_j = [(M_i^1 \cup M_j^1),\ (M_{ij}^{s_0} \cup M_i^{t_0} \cup M_j^{t_0})] \tag{3a}$$

Similarly,

$$M_i \cap M_j = [(M_i^1 \cap M_j^1),\ (M_i^0 \cup M_j^0)] = [(M_{ij}^{s_1} \cup M_i^{t_1} \cup M_j^{r_1}),\ (M_i^0 \cup M_j^0)] \tag{3b}$$

$$\overline{M}_i = [\overline{((M_i^1),\ (M_i^0))}] = [(M_i^0),\ (M_i^1)] \tag{3c}$$

2. Determination of the power of the state sets

In the above technique (as well as in determining some other criteria for the realization of these functions) the power of some subsets of states is to be found. The characteristic functions of these subsets can be described in an arbitrary form.

For this purpose [2] offers techniques for the transformation of an arbitrary Boolean expression into some "canonical" form enabling the computations of powers of various state subsets as a sum and product of the powers of the state subsets which correspond to separate parts of the function analyzed, thus significantly simplifying the computations. The use of the analytical form of the functions permits one to take full account of the information contained in the state table which corresponds to the analyzed function with no need to construct the table itself.

Let us enumerate the parentheses denoting by 1 the outer parentheses of the parenthetic expression of the Boolean function and increasing the index with the rank of the parenthesis. The subfunction in the i-th parenthesis will be referred to as the i-th disjunctive or conjunctive term depending on the outer logical operation of this subfunction (i.e., depending on the sign of the $(i+1)$ st terms contained in the expression). Inversion over the expressions will be denoted by square parenthesis and similarly enumerated.

A canonical parenthetic form which may be used to find the number of states is the form where any pair of terms included into a disjunctive term is orthogonal and all the terms of a conjunctive term should contain no coinciding variables.

The transfer to the canonical parenthetic form is done by means of the decomposition of a given parenthetic expression by variables using Shannon's rule. It is obvious that for the disjunctive term i of the canonical form the number of states equals the sum of the numbers of states of the $(i+1)$ conjunctive terms contained in this disjunctive term. The number of states of the conjunctive term will be $\alpha_i = 2^{n-k}\beta_1\beta_2\ldots\beta_m$ where n is the total number of variables, k is the number of variables contained in the conjunctive term, $\beta_1, \beta_2, \ldots, \beta_m$ is the sum of the numbers of states of the disjunctive terms contained in the conjunctive term. A term with square parenthesis (inversion) has the number of states defined as

$$\beta_j = 2^r - \beta_j^*$$

where r is the total number of variables contained in this inversion term and β_j^* is the number of states of this term.

This follows from the fact that the power of the sets of states, characterized by the inversion function is equal to addition up to 2^r (r — is the number of the variables of this function) from the power of the sets of states, characterized by function, wich is under the symbol of inversion.

Let the function

$$F = [x_i x_j \vee \bar{x}_j x_k x_n].$$

be given.

The number of states, characterised by the function, which is inside of square parenthesis (under symbol of inversion) is: $\beta^* = 6$. The number of variables of this function is: $r = 4$. Therefore the number of states, characterized by the given function is: $N_F = 2^4 - 6 = 10$.

Let us have a certain function specified by its permit (F^1) and don't care ($F^{\sim}$) states

$$F^1 = x_5 \vee \bar{x}_1 x_5 x_6 \vee x_1(\bar{x}_5 x_6 \vee x_2 \bar{x}_5 \bar{x}_6 x_{10} x_{11}) \vee x_2 x_1(\bar{x}_3 \vee \bar{x}_4 \vee \bar{x}_7 \vee \bar{x}_8 \vee \bar{x}_9),$$

$$F^{\sim} = x_3 x_4 \vee \bar{x}_8 \bar{x}_9 .$$

Obviously the functions, characterized by the sets of permit and forbid states without the don't care ones ($F^{1\sim}$ and $F^{0\sim}$) will be described as

$$F^{1\sim} = F^1 \overline{F^{\sim}} = \big(x_5 \vee \bar{x}_1 x_5 x_6 \vee x_1(\bar{x}_5 x_6 \vee x_2 \bar{x}_5 \bar{x}_6 x_{10} x_{11}) \vee$$

$$\vee x_2 x_1(\bar{x}_3 \vee \bar{x}_4 \vee \bar{x}_7 \vee \bar{x}_8 \vee \bar{x}_9)\big)[x_3 x_4 \vee \bar{x}_8 \bar{x}_9],$$

$$F^{0\sim} = \bar{F}^1 \bar{F}^{\sim} = [x_5 \vee \bar{x}_1 x_6 x_6 \vee x_1(\bar{x}_5 \bar{x}_6 \vee x_2 \bar{x}_5 x_6 x_{10} x_{11}) \vee$$

$$\vee x_2 x_1(\bar{x}_3 \vee \bar{x}_4 \vee \bar{x}_7 \vee \bar{x}_8 \vee \bar{x}_9)][x_3 x_4 \vee \bar{x}_8 \bar{x}_9].$$

Transform these expressions to canonical form using Shannon's rule in order variable: x_1, x_2, x_3, x_4.* Denoting the upper index of parenthesis by the number of states in the form $2^{n-k}\beta_1\beta_2\ldots\beta_m$ and the lower index by rank of parenthesis, we shall have:

$$F^1 = {}_1\big({}_2(^{2^8}x_1 x_2 x_3 \bar{x}_4\,{}_3[^{2^4}\bar{x}_8 \bar{x}_9]_3^{2^2\cdot 3})_2 \vee {}_2(^{2^8}x_1 x_2 \bar{x}_3\,{}_3[^{2^5}\bar{x}_8 \bar{x}_9]_3)_2^{2^3\cdot 3} \vee {}_2\big(^{2^8}x_1 \bar{x}_2\,{}_3(^{2^6}x_5 \vee$$

$$\vee \bar{x}_5 x_6)_3^{2^4\cdot 3}\,{}_3(\bar{x}_3 \vee x_3 \bar{x}_4)_3^{2^2\cdot 9}\,{}_3[\bar{x}_8 \bar{x}_9]_3)_2^{2^0\cdot 27} \vee {}_2\big(^{2^8}\bar{x}_1\,{}_3(^{2^7}x_5 \vee \bar{x}_5 x_6)_3^{2^5\cdot 3}\,{}_3(\bar{x}_3 \vee$$

$$\vee x_3 \bar{x}_4)_3^{2^3\cdot 9}\,{}_3[\bar{x}_8 \bar{x}_9]_3)_2^{2^1\cdot 2^8}\big)_1$$

$$F^0 = {}_1\big(^{2^8}_2(x_1 \bar{x}_2\,{}_3[^{2^6}x_5 \vee \bar{x}_5 x_6]_3^{2^4\cdot 1}\,{}_3(\bar{x}_3 \vee x_3 \bar{x}_4)_3^{2^2\cdot 3}\,{}_3[\bar{x}_8 \bar{x}_9]_3)_2^{2^0\cdot 9} \vee {}_2\big(^{2^8}\bar{x}_1\,{}_3[^{2^7}x_5 \vee$$

$$\vee \bar{x}_5 x_6]_3^{2^5\cdot 1}\,{}_3(\bar{x}_3 \vee x_3 \bar{x}_4)_3^{2^3\cdot 3}\,{}_3[\bar{x}_8 \bar{x}_9]_3)_2^{2^1\cdot 9}\big)_1 .$$

---

* For determining the order of variables, which give the expression, approaching the smallest amount of letters, it is useful to apply the heuristic criterion (5) or (6) (see page 7 and 8), as statistical experiments show.

Thus the powers of the sets of permit and forbid states for the given function will be

$$N^1 = 2^2 \cdot 3 + 2^3 \cdot 3 + 2^0 \cdot 27 + 2^1 \cdot 27 = 117$$

$$N^0 = 2^0 \cdot 9 + 2^1 \cdot 9 = 27.$$

3. Decomposition of Boolean functions

Realization of a given Boolean function in given elements is essentially a problem of decomposing this function into subfunctions in accord with the logical properties of the element. Obtaining the accurate solution for a problem of minimizing a Boolean function, or transformation to the form with the smallest number of operations and letters is a complex problem of combinatorial search [3, 4]. With the number of input variables as high as 20 or 30 the problem becomes hardly solvable even on computers. Therefore presently minimization of Boolean functions is achieved by means of heuristic methods with local optimization which we call the "directional search".

One of the first attempts to eliminate combinatorial search was introduced in [5] and widely used afterwards. This was the procedure of finding additional letters of the terms which describe the function in a contradictory way (the so-called "insufficient minterms"). Further in [6] a method of directional search was suggested for the case when a Boolean function was given by its table of states. The method contained criteria for selecting the so-called "inessential" variables* and finding minimal terms of the kernel as well as the minimal set of insufficient minterms.**

The fact that the function should be specified by its table of states significantly limits, however, the dimensionality of such problems. Ref. [7] suggested a technique in which minimization procedure rests upon the record of the given function and all its intermediate forms obtained in the course of minimization in an arbitrary analytical form thus considerably increasing the dimensions of the problems.

A more general technique was developed afterwards for realization of a function or a system of functions using "arbitrary" elements, or those whose logical properties are described by arbitrary Boolean functions [8].

The first stage of this technique implies elimination of the so-called "inessential" variables i.e. such whose elimination from F^1 and F^0 does not change the values of the function.

To determine inessential variables, a notion of Boolean "derivative" is used, introduced in Ref. [9]. The derivative of the given function with respect to an inessential variable is equal to zero.

$$\frac{dF}{dx_k} = F^1_{(x_k=1)} F^0_{(x_k=0)} \vee F^1_{(x_k=0)} F^0_{(x_k=1)} \tag{4}$$

* An inessential variable is a variable for which no pair of permit and forbid states exist differing only by the value of this variable. Elimination of this variable does not change the value of the functions. If a pair of permit and forbid states differs by the value of one variable, the values of this variable in these states are called obligatory letters.

** Minterm of the kernel is the conjunction of obligatory letters which describe only a subset of permit states or only a subset of forbid ones. Such terms should be included into all d-n. f. versions of a given function.

To obtain optimal realization* the order of elimination of inessential variables is important. A heuristic criterion is used for this purpose which estimates the proximity between the variable and the constant

$$R_k = n^1_{1,k} n^0_{0,k} + n^1_{0,k} n^0_{1,k} \tag{5}$$

where $n^1_{1,k}$ and $n^1_{0,k}$ are the number of permit states in the function in which the variable x_k takes on the values of 1 and 0, respectively, and $n^0_{1,k}$ and $n^0_{0,k}$ is the same for borbid states.

This criterion gives exact results in utmost cases, when the variable x_k is constant or a given function equal to the letter x_k or $\bar{x}_k$.

In the first case $n^1_{0,k}=n^0_{0,k}=0$ or $n^1_{1,k}=n^0_{1,k}=0$ and therefore $R=0$. In the second case $n^1_{0,k}=n^0_{1,k}=0$ or $n^1_{1,k}=n^0_{0,k}=0$. It is possible to show that in these cases $R=\max$.

First an inessential variable is eliminated for which we have the least value of the criterion R. After the variable is eliminated from the function F, the values of R are recomputed and the next variable is eliminated until all the variables left are essential.

Let us assign as the inputs $y_1, y_2, \ldots, y_q$ of the output element φ a certain set of input variables $x_i, x_j, \ldots, x_q$. At the output of the element we shall have then the functions h and g.**

Then it is clear that if

$$F^1\bar{h} = 0 \quad F^0\bar{g} = 0$$

the function can be realized by a single element with a given assignment of variables as inputs of this element. If these expressions are not equal zero, the realization of the function will be contradictory, i.e., for some states from M^1 "0" will appear at the output of φ the element, and for some states, from M^0, "1".

Two problems arise here:

a) find a set of variables assigned as the inputs of the output element such that the functions h and g be as proximate as possible to the functions F^1 and F^0, that provides optimization of the entire structure, and

b) design the "additional" functions with the minimal necessary number of states assigned as inputs of the output element for elimination of contradictions.

The first problem is solved by the calculation of the value of the heuristic criterion for every variable x_k

$$b_k = \frac{n^1_{1,k}}{N^1} - \frac{n^1_{0,k}}{N^0} \tag{6}$$

where: $n^1_{1,k}$ and $n^1_{0,k}$ — have the same sense, as in criterion (5); N^1 is the power of the set M^1 and N^0 is the power of the set M^0. If b_k is positive then x_k is without

* By an optimal realisation we understand the obtaining of a function, nearing to such one, which has a minimal number of variables.

** The functions h and g specify the states which in the function realized by the element are permit and forbid states, respectively.

the sign of inversion. If b_k is negativ then x_k is with the sign of inversion. The variable x_k is selected with the maximal value of b_k.

The second problem is solved by determination of the sets of permit and forbid states of the so-called "additional" function, i.e., such function by the replacement of which variable x_k, will remove or decrease contradiction in realization of the given function.

Design of these functions for an arbitrary element is a rather bard task achieving which illustrate well the problem of isolating, from the sets of states of the given function, certain subsets with given properties, which was mentioned at the beginning of this paper.

Let us examine in a more detailed manner what states have to be permit and forbid for this function.

Let us introduce notion of "partial" derivative and "ranks" of partial derivative variables.

The partial derivatives of first rank for the variable x_k are:

$$\partial^1(F)_{x_k} = F^1_{(x_k=1)} F^0_{(x_k=0)} \tag{7a}$$

and

$$\partial^1(F)_{\bar{x}_k} = F^1_{(x_k=0)} F^0_{(x_k=1)} \tag{7b}$$

Expression (7a) characterizes the set of permit states in which variable $x_k=1$ and x_k is essential. Accordingly in the set of forbid states $x_k=0$ and x_k is also essential.

Expression (7b) characterizes the set of permit states in which the variable $x_k=0$ and x_k is essential. Accordingly in the set of forbid states $x_k=1$ and x_k is essential.

In these cases, for every state from M^1 and M^0 there will be found accordingly exactly one state in M^0 and M^1 which differs by the significance of the variable x_k from the given state.

Let us consider that in these cases these states are in distance "one".

Let us understand as derivatives of rank of "j" ($\partial^j(F)_{x_k}$ and $\partial^j(F)_{\bar{x}_k}$ functions which characterise the states from F^1 or F^0 having in ratio to given state j variables (including x_k), which have opposite significance. Let us consider, that in these cases these states are in distance "j".

For the analysis of states included in additional functions the expressions

$$S(F^1) = \bigvee_{j=2}^{n} \partial^j(E)_{x_k} \tag{8a}$$

and

$$S(F^0) = \bigvee_{j=2}^{n} \partial^j(F)_{\bar{x}_k} \tag{8b}$$

will be useful.

These expressions characterise disjunctions of states for the partial derivatives of all ranks, without the first.

To obtain the expressions $S(F^1)$ and $S(F^0)$ one should consider those for each rank of partial derivatives and then join them up.

Let us consider a technique of finding second rank partial derivatives. First find the functions $D^1(F^1)$ and $D^1(F^0)$ describing the "remaining" states in M^1 and M^0 after elimination of the states included in the first rank partial derivatives.

$$D^1(F^1) = F^1\overline{\tau^1(\varphi^1)}$$

$$D^1(F^0) = F^0\overline{\tau^0(\varphi^0)}$$

where

$$\tau^1(\varphi^1) = \Sigma(y_i\partial^1(\varphi^1)_{y_i}) \vee \Sigma(\bar{y}_i\partial(\varphi^1)_{\bar{y}_i})$$

and

$$\tau^1(\varphi^0) = \Sigma(\bar{y}_i\partial^1(\varphi^1)_{y_i}) \vee \Sigma(y_i\partial(\varphi^1)_{\bar{y}_i}).$$

The second rank partial derivatives are those of the first rank for $D^1(F^1)$ and $D^1(F^0)$ over $y_i, \bar{y}_i$ with respect to $\tau^1(\varphi^1)$ and $\tau^1(\varphi^0)$.

Higher rank partial derivatives are determined in a similar way.

Let functions F^1 and F^0 which characterise permit and forbid states M^1 and M^0 for some function F be given. Let an element φ be also given, having q inputs: $y_1, y_2, \ldots, y_q$. Permit and forbid states of this element are characterized by functions: $\varphi^1 = r(y_1, y_2, \ldots, y_q)$ and $\varphi^0 = s(y_1, y_2, \ldots, y_q)$. In addition, let the set of variables: $x_i, x_j, \ldots, x_k$ be determined as assigned by inputs of element φ, which result realisation of functions: $h(x_i, x_j, \ldots, x_k)$ and $g(x_i, x_j, \ldots, x_k)$ on the output of this element, accordingly with permit and forbid of given function F, but realize it contradictory.

In [10] the following formulas are given characterising permit ($f^1_{y_i}$) and forbid ($f^0_{y_i}$) states of additional function on input y_i of element

$$f^1_{y_i} = F^1(\partial^1(h)_{y_i} \vee S(g)_{\bar{y}_i}) \vee F^0(\partial^1(h)_{\bar{y}_i} \vee S(h)_{\bar{y}_i}) \tag{9a}$$

$$f^0_{y_i} = F^1(\partial^1(h_{\bar{y}_i}) \vee S(g)_{y_i}) \vee F^0(\partial^1(h)_{y_i} \vee S(h)_{y_i}). \tag{9b}$$

Let us show, that these formulas reflect category of states including in additional function correct and completely. To the set of permit states belong follows:

a) The states in which $x_k = 1$ or $x_k = 0$ and the given function is realized correctly at the output of the element, and the change of the value x_k changes the output value which becomes contradictory. It is clear that these values should be preserved in the additional function which provide for the replacement of the variable x_k.

b) The states in which x_k also is either "1" or "0" but the function F is realised at the output of the element contradictory and the change of the value x_k leads to elimination of the contradiction. Here as in the previous case the letter x_k is an obligatory letter and in order to eliminate the contradiction the state should be replaced with the one from the opposite set of states.

Functions characterising states of categories of a) and b) will be expressed therefore in the following form

$$\Delta^1(f^1_{y_i}) = F^1\partial^1(h)_{y_i} \vee F^0\partial^1(h)_{\bar{y}_i}.$$

If the states of the element φ, leading to contradictory realisation, belong only to categories a) and b), then the function $\Delta^1(f^1_{y_i})$ completely eliminates the contradictions.

c) If the state under consideration differs from those of the opposite set of the table by values of several variables, the change of the values x_k via additional function is still helpful, since it decreases the "distance" between the given state and the one which correctly realises the given function thus simplifying the realization of additional functions at the other inputs of the element.

The function, characterising these states, will be expressed in following form

$$\Delta''(f^1_{y_i}) = F^1 S(g)_{\bar{y}_i} \vee F^0 S(h)_{\bar{y}_i}.$$

a) The states, in which correct realisation of permit states of a given function F is provided write help of other variables and therefore the change of the value of given variable don't change the significance of the output of the element, belong to don't care states of additional function.

The disjunction: $\Delta'(f^1_{y_i}) \vee \Delta''(f^1_{y_i})$ gives formula (9a). The correctness and completeness of formula (9b) prove analogous.

Successive application of formulas (9a) and (9b) for all inputs of element φ and for received additional functions give the convergent process of elimination of contradictory in the realization of the given function F.

4. Algebraic model of a discrete device

A number of problems in the analysis of discrete devices (revealing statistical and dynamic races, reliability analysis, determination of check and diagnosis tests, etc.) are very difficult because of the lack of adequate models which would describe in a compact way the internal structure of the device as well as its operating algorithm.

The model without this defect [10] uses the fact that introduction of each internal variable (a function of the same input variables) doubles the number of states of the function and exactly one half of them should belong to the states of $M^{\sim}$. Indeed, if we have some element for the function

$$\varphi_i = f_i(x_1, x_2, \ldots, x_n)$$

where $x_1, x_2, \ldots, x_n$ are the input variables, then the function $\Delta_i = \varphi_i \overline{f_i(x_1, x_2, \ldots, x_n)} \vee \vee \bar{\varphi} f_i(x_1, x_2, \ldots, x_n) \equiv 0$ i.e. it describes the subset of states $M^{\sim}$.

Additional internal variables associated with outputs of the elements are introduced for each l-th output of the structure of a discrete device by eliminating from M^1 and M^0 the states characterised by the functions Δ_i.

Similarly as above, let us denote the functions characterizing the subsets of states M^1 and M^0 at the l-th output of the structure containing k elements may be described as follows

$$F_l^{1,k} = F_l^1(x_1, x_2, \ldots, x_n)\bar{\psi}$$

$$F_l^{0,k} = F_l^0(x_1, x_2, \ldots, x_n)\bar{\psi}$$

where

$$\psi = \bigwedge_{i=1}^{k} (\varphi_i \overline{f_i(x_1, x_2, \ldots, x_n)} \vee \bar{\varphi}_i f_i(x_1, x_2, \ldots, x_n).$$

INSTITUTE OF CONTROL SCIENCES
MOSCOW, USSR

References

[1] Клини, С. К., Представление событий в нервных сетях и конечных автоматах, Сб. Автоматы. Изд. Иностр. литер., 1956.
[2] Копыленко В. М., Синтез структур релейных устройств при задании исходых логических функций в произвольной скобочной форме, Сб. Проектирование информационно-логических устройств для ирригации, Изд. ИЛИМ, 1973.
[3] QUINE, W., The problem of simplifying truth function, *Amer. Math. Monthly,* v. 59, No. 8, 1952.
[4] MC CLUSKEY J., Minimization of Boolean functions, *Bell System Tech. J.,* v. 35, No. 6, 1956, pp. 1417—1444.
[5] Гаврилов, М. А., Минимизация булевых функций, характеризующих релейные цепи, *Автоматика и телемеханика,* v. 20, No. 9, 1959.
[6] Гаврилов, М. А., В. М. Копыленко, Метод минимизации структур бесконтактных релейных устройств на функционально-полных наборах элементов, Сб. Теория дискретных автоматов, Изд. Зинатне, Рига, 1967.
[7] GAVRILOV, M. A., Development of the finite automata theory in the discrete circuit design, *Problems of Control and Information Theory,* (Jubilee Number) v. 4, 1975, pp. 77—96.
[8] Гаврилов, М. А., Декомпозиция комбинационных автоматов, Сб. Математические структуры, вычислительная математика, математическое моделирование, София, 1975, pp. 37—56.
[9] AKERS, S., On a theory of Boolean function, *J. Soc. Indust. Appl. Math.,* v. 7, No. 4, 1959.
[10] Базарбаева, Т. Г., З. И. Вострова, В. М. Копыленко, В. М. Якутин, Проектирование структур релейных устройств с учетом их надежности и некоторых особенностей технической реализации, Дискретные сустемы, Международный симпозиум, v. 2, Изд. Зинатне, Рига, pp. 7—15.

(Received March 9, 1978)