Structure of program runs of non-standard time

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1. Introduction

In this section the set of natural numbers is denoted by $\mathcal{N}$, and the set of Peano axioms (with $+$ and $\cdot$ only) by PA. In our point of view, a program is a finite sequence of labelled statements. The labels are (distinct) natural numbers. Each statement is either an assignment of the form "$v \leftarrow \tau$" where v is a variable symbol and τ is a term of Peano arithmetic containing operator symbols $+$ and $\cdot$ only, or is an if-statement of the form "IF χ THEN l" where χ is a quantifier free formula of PA and $l \in \mathcal{N}$ is a label. Denote by V_p the (finite) set of variable symbols occuring in the program p and let L_p be the set of the labels of the statements and $h \in \mathcal{N} \setminus L_p$ (the "halt" label). A *run* of the program p is a sequence $\langle l_i, f_i \rangle_{i \in \mathcal{N}}$, where

(i) $l_i \in L_p \cup \{h\}$ and $f_i \colon V_p \to \mathcal{N}$ is a valuation of the variables for every $i \in \mathcal{N}$;

(ii) if $l_i = h$ then $l_{i+1} = l_i$, $f_{i+1} = f_i$;

(iii) if the statement labelled by l_i is "$v \leftarrow \tau$" then

$$l_{i+1} = \begin{cases} l_i + 1 & \text{if } \ l_i + 1 \in L_p, \\ h & \text{otherwise,} \end{cases}$$

$$f_{i+1}(w) = \begin{cases} f_i(w) & \text{if } \ w \in V_p,\ w \neq v, \\ \tau[f_i] & \text{if } \ w = v; \end{cases}$$

(iv) if the statement labelled by l_i is "IF χ THEN l" then

$$l_{i+1} = \begin{cases} l & \text{if } \ l \in L_p \text{ and } \chi[f_i] \text{ is true,} \\ l_i + 1 & \text{if } \ l_i + 1 \in L_p \text{ and } \chi[f_i] \text{ is false,} \\ h & \text{otherwise,} \end{cases}$$

$$f_{i+1} = f_i.$$

The run of the program halts, if $l_i = h$ for some i [cf. 4].

It is well-known that every program can be written in this form because we have made no restriction on the content of variables [5]. Moreover, there is a straighforward way to prove partial correctness of programs of this type: assign formulas to every element of $L_p \cup \{h\}$ and prove (say, from the Peano axioms) that if the formula assigned to l_i is satisfied then after executing the statement belonging to l_i, the formula assigned to l_{i+1} will be satisfied, too. Then, if the run halts, the formula assigned to h is satisfied, i.e. the program is partially correct. This method is the so called Floyd—Hoare derivation [6].

It is easy to give a rigorous proof that if a program has a Floyd—Hoare derivation, i.e. if we may assign formulas and prove what we have to prove then the program is partially correct. But, alas, there are programs which always halt, always give the same result but have no Floyd—Hoare derivation. For example let φ be a formula of Peano arithmetic such that neither φ nor its negation are provable from PA. Our program checks whether its only input is the Gödel number of a proof of φ from PA. If it is, it prints 1, if not, i.e. if the input is either not a Gödel number of a proof or does not prove φ, it prints 0. Our program always halts because it is a decidable property to be a proof of a concrete formula, and always prints 0 because there is no proof of φ. Moreover, we can not prove this (from PA) because then we would be able to prove in PA that there is a non-provable formula, i.e. that PA is consistent, which is impossible.

This difficulty vanishes if we allow the program to operate not only on $\mathcal{N}$ but on any model of PA and to run not only through finite time but through non-standard time. This idea is behind the concept of continuous trace, it simulates the non-standard runs of programs, see [1], section 3 of [7], and [8].

2. Notation, definitions

Denote by L the set of classical first order formulas of type t, where t is the similarity type of arithmetic, i.e. it consists of "$+, \cdot, 0, 1$" with arities "2, 2, 0, 0", respectively. PA denotes the following (infinite) set of axioms:

P1 $x+1 \neq 0$

P2 $x+1 = y+1 \leftrightarrow x = y$

P3 $x+0 = x$

P4 $x+(y+1) = (x+y)+1$

P5 $x \cdot 0 = 0$

P6 $x \cdot (y+1) = (x \cdot y)+x$

P7 for all formulas φ with x as free variable

$$[\varphi(0) \wedge \forall x(\varphi(x) \to \varphi(x+1))] \to \forall x \varphi(x).$$

We will use other relation and function symbols, as e.g. $x<y$ or rem(x, y) which are definable in PA. We introduce the bounded quantifiers $(\forall x<y)\varphi(y) \leftrightarrow \leftrightarrow \forall x(x<y \to \varphi(x))$, etc., too. The following reformulation of the axiom of induction

P8 $[(\forall y < x)\varphi(y) \to \varphi(x)] \to \forall x\varphi(x)$

may be obtained from P7 substituting $\varphi(x)$ by $(\forall y<x)\varphi(y)$ [cf. 2].

The inclusion $A\subset B$ allows the sets A and B be equal.

To save space we use vector notation in place of sequence of symbols of same type. E.g. we write $\varphi(\bar{x})$ instead of $\varphi(x_1, x_2, \ldots, x_n)$, etc. The dimension of vectors is always clear from the context.

Definition. Let **A** be any model of PA with universe A. Let $\varphi(x_1, \ldots, x_n, y_1, \ldots, y_n)$ be a formula of PA so that

$$\mathrm{PA} \vdash \forall x_1 \ldots \forall x_n \exists ! y_1 \ldots \exists ! y_n \varphi(x_1, \ldots, y_n). \tag{2.1}$$

Let $\vec{q}_a=\langle q_a^1, \ldots, q_a^n\rangle$ be a sequence of length n of elements of A for every $a\in A$. We say that the sequence $\langle \vec{q}_a\rangle_{a\in A}$ is a *continuous trace* (ct in short) of φ if

$$\mathbf{A} \models \varphi(\vec{q}_a, \vec{q}_{a+1}) \quad \text{for every} \quad a\in A; \tag{2.2}$$

for every formula ψ of PA and every sequence $\vec{p}$ of elements of A

$$\mathbf{A} \models [\psi(\vec{q}_0, \vec{p}) \wedge \bigwedge_{a\in A} (\psi(\vec{q}_a, \vec{p}) \to \psi(\vec{q}_{a+1}, \vec{p}))] \to \bigwedge_{a\in A} \psi(\vec{q}_a, \vec{p}). \tag{2.3}$$

In the remaining part of this paper we fix the model **A** of PA, the formula φ, the sequence $\vec{q}_0$ and its length n. Whenever we speak about continuous traces we mean ct of φ with first element $\vec{q}_0$ in **A**.

We shall need the notion of coding function of sequences. Let rem(x, y) be the remainder when x is divided by y and define the ordered pair $\langle x, y\rangle$ as $(x+y)\cdot(x+y)+x$. Let moreover the triplet $\langle x, y, z\rangle=\langle x, \langle y, z\rangle\rangle$ and define the formulas PAIR(z), SEQ(u) and the functions LENGTH(u), ELEM(u, i) as follows.

$$\mathrm{PAIR}(z) = \forall u(u\cdot u \leqq z \wedge (u+1)\cdot(u+1) > z \to z \leqq u\cdot u+u);$$

$$\mathrm{SEQ}(u) = \mathrm{PAIR}(u) \wedge \forall x \forall y(u = \langle x, y\rangle \to \mathrm{PAIR}(y));$$

$$\mathrm{LENGTH}(u) = \begin{cases} n & \text{if } \mathrm{SEQ}(u) \text{ and } u = \langle x, y, n\rangle, \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathrm{ELEM}(u, i) = \begin{cases} \mathrm{rem}(m, 1+(i+1)\cdot b) & \text{if } \mathrm{SEQ}(u) \text{ and } u = \langle m, b, n\rangle \text{ and } i < n, \\ 0 & \text{otherwise.} \end{cases}$$

A straighforward proof shows that

$$\mathrm{PA} \vdash \mathrm{PAIR}(z) \to \exists ! x \exists ! y(z = \langle x, y\rangle)$$

$$\mathrm{PA} \vdash \forall u \exists ! n(\mathrm{LENGTH}(u) = n)$$

$$\mathrm{PA} \vdash \forall u \forall i \exists ! x(\mathrm{ELEM}(u, i) = x).$$

We say that $u\in A$ is a *sequence* if $\mathbf{A}\models \mathrm{SEQ}(u)$, its length is n if $\mathbf{A}\models \mathrm{LENGTH}(u)=n$ and its i-th element is a if $\mathbf{A}\models \mathrm{ELEM}(u, i)=a$. Note that 0 is a sequence of length 0.

The following theorem says that every sequence can be lengthened by 1 [3].

Theorem 1. $\mathrm{PA} \vdash \forall u\, \forall z\, \exists v(\mathrm{SEQ}(u) \to$
$\{\mathrm{SEQ}(v) \wedge \mathrm{LENGTH}(v) = \mathrm{LENGTH}(u) + 1 \wedge$
$(\forall i < \mathrm{LENGTH}(u))(\mathrm{ELEM}(u, i) = \mathrm{ELEM}(v, i)) \wedge$
$\mathrm{ELEM}(v, \mathrm{LENGTH}(u)) = z\})$. □

3. The result

First we prove some lemmas. We remind that $\mathbf{A}$, φ, $\vec{q}_0$ and n are fixed.

Lemma 1. There are a formula Φ of PA and a unique sequence $\langle \vec{q}_a \rangle_{a \in A}$ of sequences of length n of elements of A with the given $\vec{q}_0$ such that

$$\mathrm{PA} \vdash \forall m\, \forall x_1 \ldots \forall x_n\, \exists!\, y_1 \ldots \exists!\, y_n \Phi(m, x_1, \ldots, x_n, y_1, \ldots, y_n) \tag{3.1}$$

$$\mathrm{PA} \vdash \forall m\, \forall \vec{x}\, \forall \vec{y}\, \forall \vec{z}(\Phi(m, \vec{x}, \vec{y}) \wedge \Phi(m+1, \vec{x}, \vec{z}) \to \varphi(\vec{y}, \vec{z})) \tag{3.2}$$

$$\mathbf{A} \vDash \Phi(a, \vec{q}_0, \vec{q}_a) \quad \text{for every} \quad a \in A \tag{3.3}$$

$$\mathbf{A} \vDash \varphi(\vec{q}_a, \vec{q}_{a+1}) \quad \text{for every} \quad a \in A. \tag{3.4}$$

Proof. Let $\Phi_1(m)$ be

$$\forall x_1 \ldots \forall x_n\, \exists u \chi(x_1, \ldots, x_n, u, m)$$

where χ is "u is a sequence of length $m+1$ such that every element of u is a sequence of length n, the elements of the 0-th element of u are $x_1, \ldots, x_n$ in this order and for every $i < m$ the i-th element $\vec{y}$ and the $(i+1)$-st element $\vec{z}$ of u satisfy $\varphi(\vec{y}, \vec{z})$".

It is clear that $\mathrm{PA} \vdash \Phi_1(0)$, one only have to use Theorem 1 n times. In view of (2.1) and Theorem 1, $\mathrm{PA} \vdash \Phi_1(m) \to \Phi_1(m+1)$ holds. Therefore, by the induction axiom, $\mathrm{PA} \vdash \forall m \Phi_1(m)$. A very similar argument shows that the following formula, denoted by Φ_2, is also PA provable (by induction on i):

$\forall x_1 \ldots \forall x_n\, \forall u\, \forall v\, \forall i$ ("if u and v are sequences as above and $i \leq \min(\mathrm{LENGTH}(u), \mathrm{LENGTH}(v))$ then the elements of the i-th element of u and v coincide").

Now let Φ be $\exists u(\chi(x_1, \ldots, x_n, u, m) \wedge$ "the elements of the m-th element of u are $y_1, \ldots, y_n$ in this order"). The existence in (3.1) is ensured by Φ_1, the uniqueness is by Φ_2. (3.2) is trivial. Consider now the valuation of Φ in $\mathbf{A}$ where the values of $x_1, \ldots, x_n$ are $q_0^1, \ldots, q_0^n$, respectively while m has the value $a \in A$. Denote the values of $y_1, \ldots, y_n$ for which $\Phi(m, \vec{x}, \vec{y})$ holds in $\mathbf{A}$ by $q_a^1, \ldots, q_a^n$, respectively. The $\vec{q}_a$'s are determined uniquely by (3.1) and (3.3) is satisfied by definition. (3.4) follows immediately from (3.2). □

Definition. The sequence $\langle \vec{q}_a \rangle_{a \in A}$ defined previously is called the *standard continuous trace* (sct in short). (We will see later that this sequence forms a continuous trace indeed.)

Lemma 2. Let ψ be any formula of PA, $\vec{p}$ a fixed sequence of elements of A and

$$\mathbf{A} \nvDash \psi(a, \vec{q}_a, \vec{p})$$

for some $a \in A$. Then there is a least suffix with this property, i.e. $a \in A$ such that

$$\mathbf{A} \not\models \psi(a, \vec{q}_a, \vec{p}) \quad \text{but} \quad \mathbf{A} \models \psi(b, \vec{q}_b, \vec{p}) \quad \text{if} \quad b < a.$$

Proof. Suppose the contrary, i.e. whenever $\mathbf{A} \models \psi(b, \vec{q}_b, \vec{p})$ for every $b<a$ then $\mathbf{A} \models \psi(a, \vec{q}_a, \vec{p})$. Denote by $\Psi(m, \vec{x}, \vec{z})$ the formula $\exists! \vec{y}(\Phi(m, \vec{x}, \vec{y}) \wedge \wedge \psi(m, \vec{y}, \vec{z}))$. Then, by the reformulation of the induction axiom,

$$\mathrm{PA} \vdash [(\forall n < m) \Psi(n, \vec{x}, \vec{z}) \to \Psi(m, \vec{x}, \vec{z})] \to \forall m\, \Psi(m, \vec{x}, \vec{z}).$$

Now valuate this in $\mathbf{A}$ putting $\vec{q}_0$ instead of $\vec{x}$ an $\vec{p}$ instead of $\vec{z}$. Notice, that the implication in the square bracket holds, therefore the second half of the implication holds also, i.e. $\mathbf{A} \models \psi(a, \vec{q}_a, \vec{p})$ for every $a \in A$, which is a contradiction. □

Corollary. In virtue of this lemma, we may use induction type proofs for the sequence $\langle \vec{q}_a \rangle_{a \in A}$. □

Lemma 3. Let $u, v \in A$, $0<v$. If $\vec{q}_u = \vec{q}_{u+v}$ then

$$\vec{q}_{u+x} = q_{u+x+v \cdot y}$$

for every $x, y \in A$.

Proof. Let $y=1$ and use induction by the Corollary on x. After this fix x and use induction on y. □

Lemma 4. There exists an $E \subset A$ such that

if $b_1, b_2 \in E, b_1 \neq b_2$ then $\vec{q}_{b_1} \neq \vec{q}_{b_2}$; (3.5)

for every $a \in A$ there exists $b \in E$ such that $\vec{q}_a = \vec{q}_b$; (3.6)

either $E=A$ or for some $e \in E, E=\{a \in A\colon a \leqq e\}$. (3.7)

Proof. If all of the elements of the sequence $\langle \vec{q}_a \rangle_{a \in A}$ are different, the set $E=A$ satisfy (3.5)—(3.7). If not, there is an $a \in A$ so that $\vec{q}_a$ occurs at least twice in the sequence. This property is expressible by an L-formula, so, by Lemma 2, we can assume that this a is the minimal one, i.e. $\vec{q}_b$ is unique if $b<a$. There are other occurrences of $\vec{q}_a$, hence, also by Lemma 2, there is a second one, i.e. there is an $e \geqq a$ such that $\vec{q}_a = \vec{q}_{e+1}$ but $\vec{q}_a \neq \vec{q}_b$ if $a<b\leqq e$. We claim that the set $E=\{a \in A\colon a \leqq e\}$ satisfies (3.5) and (3.6). It is sufficient to see (3.5) in case $a<b_1< <b_2 \leqq e$ only. Suppose $\vec{q}_{b_1} = \vec{q}_{b_2}$. Lemma 3 with the cast $u=b_1, v=b_2-b_1$, $x=\mathrm{rem}\,(e+1-b_1, b_2-b_1)$ gives $\vec{q}_{e+1} = \vec{q}_{b_1+x} \neq \vec{q}_a$ which is a contradiction. (3.6) is an easy consequence of Lemma 3. □

Now we have all of the tools for the proof of the main result of this paper. First we need some more preliminaries.

Definition. The subset $S \subset A$ is a *slice* if $a \in S$ implies $a+1 \in S$ and $b \in S$ for all $b<a$.

The subset $T \subset A$ is a *thread* if $a \in T$ implies, $a+1 \in T$, $a-1 \in T$ (for $a \neq 0$) and $a, b \in T$, $a<b$ imply that for some natural number n

$$b = a+1+\ldots+1 \quad (n \text{ times}).$$

The function $f: A \to A$ is a *projector* if $f(0)=0$ and $f(a+1)=f(a)+1$ for every $a \in A$.

The sequence $\langle \vec{p}_a \rangle_{a\in A}$ is a *projection* of the sequence $\langle \vec{q}_a \rangle_{a\in A}$ if there exists a projector f and a slice S such that

$$\vec{p}_a = \vec{q}_{f(a)} \text{ for every } a \in A \tag{3.8}$$

$$\mathrm{Rng}(f) \subset S \tag{3.9}$$

$$\text{for every } b \in S \text{ there is an } a \in A \text{ such that } \vec{q}_b = \vec{q}_{f(a)}. \tag{3.10}$$

Theorem 2. The standard continuous trace $\langle \vec{q}_a \rangle_{a\in A}$ forms a continuous trace.

Proof. (2.2) of the definition is satisfied by (3.4) of Lemma 1. (2.3) is immediate from the Corollary of Lemma 2. □

Theorem 3. The projections of the standard continuous trace are continuous traces. Moreover every continuous trace is a projection of the standard one.

REMARKS. An easy consequence of Theorem 3 is that if **A** is a non-standard model of PA then for all φ and $\vec{q}_0$, the cardinality of ct's of φ with first element $\vec{q}_0$ is $2^{|A|}$.

Now consider a ct $\langle \vec{p}_a \rangle_{a\in A}$ the defining formula of which is

$$\varphi(x_1, \ldots, x_n, y_1, \ldots, y_n) = \text{“} y_1 = x_1 + 1 \wedge \ldots \text{”}$$

and let $p_0^1 = 0$ for the initial position $\vec{p}_0$. Then $p_{a+1}^1 = p_a^1 + 1$ for every $a \in A$ but one can not hope for $p_a^1 = a$ in general. Actually, by Theorem 3, the standard ct is the only ct which has this property. We may interpret this phenomenon as follows. We add a “clock” to a continuous trace and suppose that the clock works well (i.e. it jumps by 1 at every step). If we require the clock to show the correct time then the ct is unique. Compare with Theorem 3.3 of [7].

Proof of the theorem. First let $f: A \to A$ be the projector and S the slice of a projection. We prove that $\langle \vec{p}_a \rangle_{a\in A}$ is a ct. (2.2) of the definition follows from $f(a+1)=f(a)+1$. To prove (2.3) let $\psi \in \mathrm{L}$ be arbitrary and assume

$$\mathbf{A} \models \psi(\vec{p}_0, \vec{p}) \wedge \bigwedge_{a\in A} (\psi(p_a, \vec{p}) \to \psi(\vec{p}_{a+1}, \vec{p})).$$

By the hypotheses, $\vec{p}_0 = \vec{q}_0$, for all $b \in S$ there is an $a \in A$ such that $\vec{q}_b = \vec{p}_a$ and if $\vec{q}_b = \vec{p}_a$ then $\vec{q}_{b+1} = \vec{p}_{a+1}$. So we know that

$$\mathbf{A} \models \psi(\vec{q}_0, \vec{p}) \quad \text{and} \quad \mathbf{A} \models \psi(\vec{q}_a, \vec{p}) \to \psi(\vec{q}_{a+1}, \vec{p}) \quad \text{for all} \quad a \in S \tag{3.11}$$

and it is enough to show that this implies

$$\mathbf{A} \models \psi(\vec{q}_a, \vec{p}) \quad \text{for all} \quad a \in S.$$

Suppose the contrary. Then, by Lemma 2, there is a least counter-example, say b. But this b belongs to S because S is a slice, which contradicts (3.11).

Now we turn to the second and longer part of the proof. Let $\langle \vec{p}_a \rangle_{a\in A}$ be any ct, $\vec{p}_0 = \vec{q}_0$. We claim that for any $a \in A$ there exists a least $b \in A$ such that $\vec{p}_a = \vec{q}_b$. Indeed, let $\psi(\vec{y}, \vec{x})$ be “$\exists m \Phi(m, \vec{x}, \vec{y})$”. It is clear that $\mathbf{A} \models \psi(\vec{p}_0, \vec{q}_0)$ with $m=0$ and if $\mathbf{A}\models\psi(\vec{p}_a,\vec{q}_0)$ with some m, then $\mathbf{A}\models\psi(\vec{p}_{a+1},\vec{q}_0)$ with $(m+1)$ because the successors are unique. Then by (2.3), $\mathbf{A}\models\psi(\vec{p}_a,\vec{q}_0)$ for all $a\in A$ which states the existence of b. Finally, Lemma 2 ensures a least one. Denote this b by $f(a)$ and denote S the range of the function f. Let moreover E and e be as defined in Lemma 4.

It is clear that $S\subset E$, $f(0)=0$ and if $f(a)\neq e$ then $f(a+1)=f(a)+1$. Now let $b\in A$, $b\notin S$ and let $\psi(\vec{x},\vec{y},m)$ be the formula "$(\exists m'<m)\Phi(m',x,y)$". Since $0<b$ and if $f(a)<b$ then $f(a+1)\leqq f(a)+1<b$, we know

$$\mathbf{A}\models\psi(\vec{p}_0,\vec{q}_0,b)\quad\text{and}\quad \mathbf{A}\models\psi(\vec{p}_a,\vec{q}_0,b)\to\psi(\vec{p}_{a+1},\vec{q}_0,b).$$

By (2.3) of the definition of ct, $\mathbf{A}\models\psi(\vec{p}_a,\vec{q}_0,b)$, i.e. $f(a)<b$ for all $a\in A$. This means that if $b\in S$ and $c<b$ then $c\in S$.

We distinguish two cases.

1. $E=A$ or $E\neq A$ but $e\notin S$. In this case f is a projector and S is a slice, therefore $\langle\vec{p}_a\rangle_{a\in A}$ is a projection.

2. $e\in S$, i.e. $S=E$. Let $b\in E$ so that $\vec{q}_b=\vec{q}_{e+1}$. By Lemma 3 $\vec{q}_u=\vec{q}_{e+1}=\vec{q}_b$ if $u=a+y\cdot(e+1-a)$, so if we choose y large enough then the thread T_u of u does not contain b. For each thread T define the function g_T: $T\to A$ as follows. If T is the thread of 0 then let $g_T(a)=a$. Otherwise if $f(v)=b$ for some $v\in T$ then let $g_T(a)=b+a-v$. Otherwise if $f(v)=e$ for some $v\in T$ then let $g_T(a)=u-1+a-v$, otherwise let $g_T(a)=f(a)$. Finally, let $g(a)=g_T(a)$ if a is in the thread T.

It is clear from the definition of g_T that g is a projector and $\vec{p}_a=\vec{q}_{f(a)}=\vec{q}_{g(a)}$. By Lemma 4, every $\vec{q}_b$ equals to some $\vec{p}_a$, i.e. in this case the projector g and the whole A as a slice shows that $\langle\vec{p}_a\rangle_{a\in A}$ is a projection. $\square$

Abstract

Continuous traces are introduced to simulate program runs when time is measured by the elements of a non-standard model of Peano axioms. This concept is a very useful one in considerations of program verification. We give here a full description of continuous traces in every model of PA. It turns out that there is exactly one continuous trace definable by a formula of PA and every other one can be got from this by a simple transformation.

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