On the role of blocking in rewriting systems

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Introduction

A rewriting system G generates a set of sentential froms $\underline{\text{sent}}\ G$ (see, e.g., [9]). If G is "pure" (see. e.g., [5]), i.e. it does not use nonterminals, then $\underline{\text{sent}}\ G$ forms also the language of G, denoted $L(G)$. In this sense every sentential form of G is successful. If G is not pure, i.e. it uses nonterminals, then the language of G consists of only those sentential forms that do not contain nonterminal symbols. In this case a sentential form is (potentially) successful if it can be rewritten (perhaps in a number of steps) into an element of $L(G)$.

Thus, naturally, $\underline{\text{sent}}\ G$ gets divided into "blocking" and "nonblocking" (hence successful) sentential forms.

The possibility of having blocking sentential forms in a grammar is often useful. In a particular derivation of a word w, G may "guess" a property of a sentential form currently rewritten and if the guess was incorrect G will take care of the fact that the derivation is dead-ended. This is a typical way of programming a language through a context-sensitive grammar (see, e.g., [9]). Also the synchronization mechanism in E(T)OL systems (see for example [7] and [8]) is a typical example of the use of a blocking mechanism.

In this paper we investigate the role that this blocking mechanism plays in rewriting systems. In particular, we do this for the grammars of the Chomsky hierarchy (Section II), EOL systems (Section III) and ETOL systems (Section IV).

I. Preliminaries and basic definitions

We assume the reader to be familiar with the rudiments of formal language theory as, e.g., in the scope of [7] and [9]. In order to fix our notation we recall some basic notions now.

For a word x, $|x|$ denotes its length and $\underline{\text{alph}}\ x$ denotes the set of letters occurring in x. For a language K, $\underline{\text{alph}}\ K = \bigcup_{x \in K} \underline{\text{alph}}\ x$. The empty word is denoted by Λ.

Let Σ_1 and Σ be alphabets, such that $\Sigma_1 \subseteq \Sigma$. Then the homomorphism $\underline{\text{Pres}}_{\Sigma,\Sigma_1}$ from Σ^* into Σ_1^* is defined as follows. If $a \in \Sigma_1$, then $\underline{\text{Pres}}_{\Sigma,\Sigma_1} a = a$ and if $a \in \Sigma \setminus \Sigma_1$, then $\underline{\text{Pres}}_{\Sigma,\Sigma_1} a = \Lambda$. To avoid cumbersome notation we often write $\underline{\text{Pres}}_{\Sigma_1}$ instead of $\underline{\text{Pres}}_{\Sigma,\Sigma_1}$, whenever Σ is understood from the context.

The mapping $\underline{\text{mir}}$ from Σ^* into Σ^* is defined by: if $w=xy$, with $x\in\Sigma^*$ and $y\in\Sigma$, then $\underline{\text{mir}}\, w=y\, \underline{\text{mir}}\, x$; $\underline{\text{mir}}\, \Lambda=\Lambda$.

Definition I.1. (i) A *grammar* is an ordered quadruple $G=(V, \Sigma, P, S)$, where V is a finite non-empty alphabet, the *total alphabet of* G, $\Sigma\subset V$ is the *terminal alphabet of* G, $V\setminus\Sigma$ is the *nonterminal alphabet of* G, $S\in V\setminus\Sigma$ is the *axiom of* G and P is a finite subset of $V^*(V\setminus\Sigma)V^*\times V^*$; the elements of P are called the *productions of* G and for $(\alpha, \beta)\in P$ we write $\alpha\to\beta$.

(ii) A word $v\in V^*$ *directly derives* a word $w\in V^*$ *according to* G, denoted $v\underset{G}{\Rightarrow}w$, if there are $x, y, \alpha, \beta\in V^*$ such that $v=x\alpha y$, $w=x\beta y$ and $\alpha\to\beta$ is a production of G. We write $x\overset{0}{\underset{G}{\Rightarrow}}x$ for every $x\in V^*$ and for $n\geqq 1$, $x\overset{n}{\underset{G}{\Rightarrow}}y$ if for some $z\in V^*$, $x\overset{n-1}{\underset{G}{\Rightarrow}}z\underset{G}{\Rightarrow}y$. We write $x\overset{+}{\underset{G}{\Rightarrow}}y$ ($x\overset{*}{\underset{G}{\Rightarrow}}y$, $x\overset{\leqq m}{\underset{G}{\Rightarrow}}y$, respectively) if $x\overset{t}{\underset{G}{\Rightarrow}}y$ for some integer $t>0$ ($t\geqq 0$, $t\leqq m$, respectively). If no confusion is possible we use, $\Rightarrow$, $\overset{+}{\Rightarrow}, \overset{*}{\Rightarrow}, \overset{n}{\Rightarrow}, \overset{\leqq n}{\Rightarrow}$ rather than $\underset{G}{\Rightarrow}, \overset{+}{\underset{G}{\Rightarrow}}, \overset{*}{\underset{G}{\Rightarrow}}, \overset{n}{\underset{G}{\Rightarrow}}, \overset{\leqq n}{\underset{G}{\Rightarrow}}$.

(iii) The set of *sentential froms* of G, denoted $\underline{\text{sent}}\, G$, is defined by $\underline{\text{sent}}\, G= =\{w\in V^*: S\overset{*}{\underset{G}{\Rightarrow}}w\}$.

(iv) The *language of* G, denoted $L(G)$ is defined by $L(G)=\{w\in\Sigma^*: S\overset{*}{\underset{G}{\Rightarrow}}w\}= =\underline{\text{sent}}\, G\cap\Sigma^*$.

Definition I.2. Let $G=(V, \Sigma, P, S)$ be a grammar.

(i) G is termed *regular*, if $\alpha\to\beta\in P$ implies $\alpha\in V\setminus\Sigma$ and $\beta\in\Sigma(V\setminus\Sigma)$ or $\beta\in\Sigma$.

(ii) G is termed *context-free*, if $\alpha\to\beta\in P$ implies $\alpha\in V\setminus\Sigma$ and $\beta\in V^+$.

(iii) G is termed *context-sensitive (monotonic)* if $\alpha\to\beta\in P$ implies $|\alpha|\leqq|\beta|$.

The families of languages generated by regular, context-free, context-sensitive and arbitrary grammars will be denoted by $\mathscr{L}$(Reg), $\mathscr{L}$(CF), $\mathscr{L}$(CS) and $\mathscr{L}$(RE) respectively.

Definition I.3. (i) An ETOL *system* is an ordered quadruple $H=(V, \Sigma, \mathscr{P}, \omega)$, where V, Σ and $V\setminus\Sigma$ are as in the definition of a grammar, $\omega\in V^+$ is the *axiom of* H and $\mathscr{P}$ is a finite non-empty set of *tables* $P_1, \ldots, P_n$, $n\geqq 1$. A table P_i, $1\leqq i\leqq n$, is a finite subset of $V\times V^*$, such that for each $\alpha\in V$ there exists a $\beta\in V^*$ with $(\alpha, \beta)\in P_i$. An element (α, β) of P_i, $1\leqq i\leqq n$, is called *a-production* and is usually written as $\alpha\to\beta$ $\cdot\alpha\to\beta$ is called an *α-production* and the fact that $\alpha\to\beta$ belongs to P_i, $1\leqq i\leqq n$, respectively to $\mathscr{P}$, is often abbreviated as $\alpha\underset{P_i}{\to}\beta$, respectively $\alpha\underset{\mathscr{P}}{\to}\beta$.

(ii) A word $v\in V^*$ *directly derives* a word $u\in V^*$ *according to* H, denoted $v\underset{H}{\Rightarrow}u$, if $v=\alpha_1\ldots\alpha_k$, $\alpha_i\in V$ for $1\leqq i\leqq k$, $u=\beta_1\ldots\beta_k$, $\beta_i\in V^*$ for $1\leqq i\leqq k$, and there exists a $j\in\{1, \ldots, n\}$ such that $\alpha_i\underset{P_j}{\to}\beta_i$, for all $i\in\{1, \ldots, n\}$. We write $x\overset{0}{\underset{H}{\Rightarrow}}x$ for every $x\in V^*$ and for $n\geqq 1$, $x\overset{n}{\underset{H}{\Rightarrow}}y$ if for some $z\in V^*$, $x\overset{n-1}{\underset{H}{\Rightarrow}}z\underset{H}{\Rightarrow}y$. We write

$x \overset{+}{\underset{H}{\Rightarrow}} y$ ($x \overset{*}{\underset{H}{\Rightarrow}} y$, $x \overset{\leqq m}{\underset{H}{\Rightarrow}} y$, respectively) if $x \overset{t}{\underset{H}{\Rightarrow}} y$ for some integer $t>0$ ($t \geqq 0$, $t \leqq m$, respectively). If no confusion is possible we use $\Rightarrow, \overset{+}{\Rightarrow}, \overset{*}{\Rightarrow}, \overset{n}{\Rightarrow}, \overset{\leqq n}{\Rightarrow}$ rather than $\underset{H}{\Rightarrow}, \overset{+}{\underset{H}{\Rightarrow}}, \overset{*}{\underset{H}{\Rightarrow}}, \overset{n}{\underset{H}{\Rightarrow}}, \overset{\leqq n}{\underset{H}{\Rightarrow}}$.

(iii) The set of *sentential forms of H*, denoted $\underline{\text{sent}}\, H$, is defined by $\underline{\text{sent}}\, H = \{v \in V^*: \omega \overset{*}{\underset{H}{\Rightarrow}} v\}$.

(iv) The *language of H*, denoted $L(H)$ is defined by $L(H) = \{v \in \Sigma^*: \omega \overset{*}{\underset{H}{\Rightarrow}} v\} = \underline{\text{sent}}\, H \cap \Sigma^*$.

Definition I.4. Let $H=(V, \Sigma, \mathcal{P}, \omega)$ be an ETOL system, with $\mathcal{P} = \{P_1, \ldots, P_n\}$.

(i) If $\mathcal{P}$ consists of one table only, say $\mathcal{P}=\{P\}$, then H is termed an EOL *system* and denoted $H=(V, \Sigma, P, \omega)$.

(ii) If, for every $\alpha \underset{\mathcal{P}}{\rightarrow} \beta$, $\beta \in V^+$, then H is termed a *propagating* ETOL system, denoted EPTOL system.

(iii) If for all $i \in \{1, \ldots, n\}$, $\alpha \underset{P_i}{\rightarrow} \beta$ and $\alpha \underset{P_i}{\rightarrow} \gamma$ implies $\beta=\gamma$, then H is termed a *deterministic* ETOL system, denoted EDTOL system.

(iv) If $\Sigma=V$, then H is termed a TOL *system*.

From the above definition it follows that we consider OL, POL, DOL, PDOL, TOL, PTOL, DTOL, PDTOL, EOL, EPOL, EDOL, EPDOL, ETOL, EPTOL, EDTOL and EPDTOL systems. The family of languages generated by X systems, where X stands for one of the above mentioned abbreviations, will be denoted by $\mathcal{L}(X)$.

Let H be an ETOL system. If the sequence $D=(x_0, x_1, \ldots, x_n)$ is such that $x_i \underset{H}{\Rightarrow} x_{i+1}$, $0 \leqq i < n$, then each occurrence of a letter in every word from $x_0, \ldots, x_{n-1}$ has a unique contribution to x_n. If A is an occurrence of a letter in x_i, $0 \leqq i < n$, then we use $\underline{\text{ctr}}_{D, x_i} A$ to denote this contribution.

Two languages, L_1 and L_2, are considered to be equal if $L_1 \cup \{\Lambda\} = L_2 \cup \{\Lambda\}$. We consider two families of languages, $\mathcal{L}_1$ and $\mathcal{L}_2$, to be equal if they differ at most by $\{\Lambda\}$. Two language generating devices G and H are said to be *equivalent* if $L(G)=L(H)$.

Definition I.5. Let $H=(V, \Sigma, P, \omega)$ be an EOL system. If there exists a subset $\Phi \subseteqq V \setminus \Sigma$ such that for all $\alpha \in \Sigma \cup \Phi$, $\alpha \underset{P}{\rightarrow} \beta$ implies $\beta \in \Phi^+$, then H is called a *synchronized* EOL system, abbreviated sEOL system. Φ is called the *set of synchronization symbols of H*.

The following result is well known, see, e.g., [3].

Lemma I.1. For every EOL system, there exists an equivalent sEOL system.

The following is the central notion of this paper.

Definition I.6. (i) A grammar $G=(V, \Sigma, P, S)$ is *nonblocking* if for every word $v \in \underline{\text{sent}}\, G$ there exists a word $u \in \Sigma^*$, such that $v \overset{*}{\underset{G}{\Rightarrow}} u$.

(ii) An ETOL system $H=(V, \Sigma, \mathscr{P}, \omega)$ is *nonblocking* if for every word $v \in \underline{\text{sent}}\, H$ there exists a word $u \in \Sigma^*$, such that $v \overset{*}{\underset{H}{\Rightarrow}} u$.

REMARK. Note that if G is a nonblocking grammar or a nonblocking ETOL system, then either $L(G) \setminus \{\Lambda\} \neq \emptyset$ or $S \overset{+}{\underset{G}{\Rightarrow}} \Lambda$ and $L(G) = \{\Lambda\}$.

The families of languages generated by nonblocking regular, nonblocking context-free, nonblocking context-sensitive, nonblocking arbitrary grammars or by nonblocking X systems (where X stands for ETOL or one of its subclasses) will be denoted by $\mathscr{L}(\text{nbReg})$, $\mathscr{L}(\text{nbCF})$, $\mathscr{L}(\text{nbCS})$, $\mathscr{L}(\text{nbRE})$ and $\mathscr{L}(\text{nb}X)$, respectively.

Lemma I.2. If $X \in \{\text{Reg, CF, CS, RE}\}$ or X stands for ETOL or one of its subclasses, then $\mathscr{L}(\text{nb}X) \subseteq \mathscr{L}(X)$.

II. The Chomsky hierarchy

In this section we impose the nonblocking condition on regular, context-free, context-sensitive and arbitrary grammars.

We start by recalling a well known fact concerning the first two types of grammars.

Lemma II.1. For every context-free (regular) grammar generating a non-empty language, there exists an equivalent nonblocking context-free (regular) grammar.

Proof. Since for every context-free (regular) grammar, there exists an equivalent context-free (regular) grammar in which every nonterminal is useful (see, e.g., [9], otherwise the generated language is empty) the lemma holds. □

Thus we get the following result.

Theorem II.1. (i) $\mathscr{L}(\text{nbReg}) = \mathscr{L}(\text{Reg})$.
(ii) $\mathscr{L}(\text{nbCF}) = \mathscr{L}(\text{CF})$.

For context-sensitive grammars generating non-empty languages we have a similar situation. However, the proof is much more involved. For this reason we give only an intuitive description of the proof. For a formal, detailed proof, we refer the interested reader to the Appendix.

Lemma II.2. For every context-sensitive grammar, generating a non-empty language there exists an equivalent nonblocking context-sensitive grammar.

Proof. Let $K \subseteq \Sigma^*$ be a non-empty language, generated by a context-sensitive grammar. We distinguish two cases.

(i) K is finite. Then, obviously, the context-sensitive grammar $(\Sigma \cup \{S\}, \Sigma, P, S)$ with $P = \{S \to x : x \in K\}$ is nonblocking and generates K.

(ii) K is infinite. Let $\Sigma' = \{[a, b, c, d] : a, b, c, d \in \Sigma\} \cup \{[a, b, c] : a, b, c \in \Sigma\} \cup \{[a, b] : a, b \in \Sigma\} \cup \{[a] : a \in \Sigma\}$; let h be the homomorphism from Σ'^* into Σ^* defined by $h([a, b, c, d]) = abcd$, $h([a, b, c]) = abc$, $h([a, b]) = ab$ and $h([a]) = a$. Let

$$K' = \{[a_1, a_2, a_3, a_4]\dots[a_{4n-3}, a_{4n-2}, a_{4n-1}, a_{4n}]: n \geqq 2, a_1\dots a_{4n} \in K\} \cup$$
$$\{[a_1, a_2, a_3, a_4]\dots[a_{4n-3}, a_{4n-2}, a_{4n-1}, a_{4n}][a_{4n+1}]: n \geqq 2, a_1\dots a_{4n+1} \in K\} \cup$$
$$\{[a_1, a_2, a_3, a_4]\dots[a_{4n-3}, a_{4n-2}, a_{4n-1}, a_{4n}][a_{4n+1}, a_{4n+2}]: n \geqq 2, a_1\dots a_{4n+2} \in K\} \cup$$
$$\{[a_1, a_2, a_3, a_4]\dots[a_{4n-3}, a_{4n-2}, a_{4n-1}, a_{4n}][a_{4n+1}, a_{4n+2}, a_{4n+3}]: n \geqq 2, a_1\dots a_{4n+3} \in K\}.$$

Clearly K' is context-sensitive, say it is generated by a context-sensitive grammar $G'=(V', \Sigma', P', S')$. Moreover $h(K')=K\setminus\{x\in K: |x|<8\}$. Now we can construct a nonblocking context-sensitive grammar $G=(V, \Sigma, P, S)$ generating K. It works as follows.

(1) $S\to x$ is in P for $x\in K$ with $|x|<8$.

(2) $P'\subseteq P$.

(3) S directly derives S' surrounded by markers. Hence K' can be derived, surrounded by these markers. A successful derivation in G terminates by rewriting elements of Σ' into elements of Σ (after it was checked by markers that a current sentential form consists of letters from Σ') and making the markers disappear. (The deletion of markers and rewriting symbols of Σ' into symbols of Σ is paired together so that the monotonicity of the productions is guaranteed).

(4) From the above it follows that $K\subseteq L(G)$.

(5) At any stage in the derivation process of a word from K' (modulo markers) a "dead" symbol N can be introduced. Then all symbols (except the leftmost and rightmost marker) in the current sentential form can (and will) eventually be replaced by N; to the right of the rightmost marker (which now also changes into N) the axiom S' of G', surrounded by markers, will be introduced again. This process may be repeated an arbitrary number of times.

(6) If from S' a word w of K' is derived, then termination can take place if w is long enough (K' is infinite!) to "absorb" all dead symbols and markers, when the symbols of Σ' are rewritten into symbols of Σ. Again, during this termination process, there still is a possibility to change all symbols of the current sentential forms into N's and to place S', surrounded by markers to the right of this string. In this case the derivation process "switches" again into state (5).

(7) Now (5) and (6) imply that $L(G)\subseteq K$, G is nonblocking and monotonic. This together with (4) implies the result. □

Corollary II.1. For every arbitrary grammar, generating a non-empty language, there exists an equivalent nonblocking grammar.

Thus we have the following result.

Theorem II.2. (i) $\mathscr{L}$(nbCS)$=\mathscr{L}$(CS).

(ii) $\mathscr{L}$(nbRE)$=\mathscr{L}$(RE).

Although it follows from Lemma II.2 that for any context-sensitive grammar, generating a non-empty language, there exists an equivalent nonblocking context-sensitive grammar, the proof of this fact was not effective; it is well known that it is not effectively decidable whether or not the language generated by a context-sensitive grammar is finite (see, e.g., [9]). Moreover, there is no algorithm which, given an arbitrary context-sensitive grammar G (generating a non-empty language) yields an equivalent nonblocking context-sensitive grammar. We also show that it is undecidable whether or not an arbitrary context-sensitive grammar G itself is nonblocking.

We prove the above two statements using Post's Correspondence Problem (see, e.g., [9]).

Definition II.1. An *instance of Post's Correspondence Problem* over an alphabet Σ is a pair (A, B), where $A=\{\alpha_1, \ldots, \alpha_n\}$, $B=\{\beta_1, \ldots, \beta_n\}$, $n \geqq 1$ with $\alpha_i \in \Sigma^+$ and $\beta_i \in \Sigma^+$, for $1 \leqq i \leqq n$. (A, B) is said to have a *solution* if there exists a non-empty finite sequence of indices $\{i_1, \ldots, i_k\}$, $i_j \in \{1, \ldots, n\}$ for $1 \leqq j \leqq k$, such that $\alpha_{i_1} \ldots \alpha_{i_k} = \beta_{i_1} \ldots \beta_{i_k}$.

Theorem II.3. There is no algorithm to decide whether or not an arbitrary instance of Post's Correspondence Problem over a two letter alphabet has a solution.

Theorem II.4. There is no algorithm that given an arbitrary context-sensitive grammar generating a non-empty language constructs an equivalent nonblocking context-sensitive grammar.

Proof. Let (A, B) be an arbitrary instance of Post's Correspondence Problem, $A=\{\alpha_1, \ldots, \alpha_n\}$ and $B=\{\beta_1, \ldots, \beta_n\}$, with $n \geqq 1$, $\alpha_i \in \{a, b\}^+$ and $\beta_i \in \{a, b\}^+$, for $1 \leqq i \leqq n$. The context-sensitive grammar G is defined as follows. $G=(V, \{c, d\}, P, S)$, where $V=\{S, Z, a, b, \overleftarrow{M}, \overrightarrow{M}, \overrightarrow{M}_a, \overrightarrow{M}_b, \overleftarrow{M}_a, \overleftarrow{M}_b, Q, N, c, d\}$ and P is given in (1) through (9).

(1) $S \to c$.
(2) $S \to c\alpha_i Z\, \underline{\mathrm{mir}}\, \beta_i c$, for $1 \leqq i \leqq n$, and $Z \to \alpha_i Z\, \underline{\mathrm{mir}}\, \beta_i$, for $1 \leqq i \leqq n$.
(3) $Z \to \overleftarrow{M} d$.
(4) $\alpha \overleftarrow{M} \to \overleftarrow{M} \alpha$, for $\alpha \in \{a, b, d\}$, and $c\overleftarrow{M} \to c\overrightarrow{M}$.
(5) $\overrightarrow{M}\alpha \to c\overrightarrow{M}_\alpha$, for $\alpha \in \{a, b\}$, and $\overrightarrow{M} d \to dQ$.
(6) $\overrightarrow{M}_\alpha \beta \to \beta \overrightarrow{M}_\alpha$, for $\beta \in \{a, b, d\}$, and $\overrightarrow{M}_\alpha c \to \overleftarrow{M}_\alpha c$, for $\alpha \in \{a, b\}$.
(7) $\alpha \overleftarrow{M}_\alpha \to \overleftarrow{M} c$, for $\alpha \in \{a, b\}$.
(8) $\beta \overleftarrow{M}_\alpha \to Nc$, for $\alpha, \beta \in \{a, b\}$ and $\alpha \neq \beta$.
(9) $Q\alpha \to Nc$, for $\alpha \in \{a, b\}$ and $Qc \to cc$.

It is rather easy to see that $L(G)=\{c\}$ if (A, B) has no solution and that $L(G)$ is infinite otherwise.

Assume that we could effectively construct an equivalent nonblocking grammar $G'=(V', \{c, d\}, P', S')$ for G. Let $n_0=\min\{|w| : S' \overset{*}{\underset{G'}{\Rightarrow}} w$ and $|w| \geqq 2\}$. Obviously we can effectively decide whether or not n_0 exists because G' is monotonic. Since G' is nonblocking, if n_0 exists then $L(G')=L(G)$ contains a word of length at least two and so (A, B) has a solution. If n_0 does not exist, then $L(G')=L(G)=\{c\}$ and hence (A, B) has no solution.

Hence if the algorithm in question exists then Post's Correspondence Problem is decidable; this contradicts Theorem II.3. □

Theorem II.5. It is undecidable whether or not an arbitrary context-sensitive grammar generating a non-empty language is nonblocking.

Proof. Let (A, B) be as in the proof of Theorem II.4. Let $H=(V, \{c, d\}, P'', S)$ be the context-sensitive grammar which is defined as follows. V and S are as in the grammar $G=(V, \{c, d\}, P, S)$ defined in the proof of Theorem II.4. P'' is defined by (1) through (8) as stated there and additionally by:

(9′) $Q\alpha \to \alpha Q$ and $\alpha Qc \to Ncc$ for $\alpha \in \{a, b\}$ and

(10) $\alpha N \to Nc$, for $\alpha \in \{a, b\}$, $dN \to Nd$ and $cN \to cc$.

Hence $L(H) \neq \emptyset$ $(c \in L(H))$ and H is nonblocking if and only if (A, B) has no solution.

Thus, if we would have an effective decision procedure for the nonblocking property of context-sensitive grammars, then Post's Correspondence Problem would be decidable. This contradicts Theorem II.3. □

We conclude this section with the following observations.

For an arbitrary grammar generating a non-empty language, there exists an effective procedure to construct an equivalent nonblocking grammar. This is a consequence of the possibility of using length-decreasing productions for the markers and the dead symbols (as used in the proof of Lemma II.2). Hence we do not need arbitrarily large words to "absorb" all those garbage symbols. Consequently, it is not needed anymore to distinguish between the case of a finite and the case of an infinite language (which made the proof of Lemma II.2 ineffective).

It is well known that it is not decidable whether an arbitrary context-sensitive grammar generates the empty language (see, e.g. [9]). Consequently it is not decidable whether or not an arbitrary context-sensitive grammar has an equivalent nonblocking context-sensitive grammar. Note that in the case of context-free grammars these questions are decidable: finiteness and emptiness are decidable for those grammars.

III. Systems without tables

We will now investigate the effect that the nonblocking condition has on the language generating power of E(P)(D)OL systems.

First we compare EOL and nbEOL systems.

It turns out that the nonblocking restriction is a real restriction. This result should be compared with the results of the previous section.

Lemma III.1. $\mathscr{L}(\text{EPOL}) \setminus \mathscr{L}(\text{nbEOL}) \neq \emptyset$.

Proof. We will prove that $K=\{a^3\} \cup \{a^{2^n}: n \geqq 0\} \in \mathscr{L}(\text{EPOL}) \setminus \mathscr{L}(\text{nbEOL})$.

(i) Let G be the EPOL system which is defined by

$$G = (\{S, A, N, a\}, \{a\}, \{S \to a^3, S \to A, A \to AA, A \to a, a \to N, N \to N\}, S).$$

Obviously $L(G)=K$. Thus $K \in \mathscr{L}(\text{EPOL})$.

(ii) The fact that $K \notin \mathscr{L}(\text{nbEOL})$ is proved by a contradiction. Assume that $K \in \mathscr{L}(\text{nbEOL})$. Then there exists a nbEOL system $H=(V, \Sigma, P, \omega)$ such that $L(H)=K$ or $L(H)=K \cup \{\Lambda\}$.

Since H is nonblocking for every $v \in K$, $v \overset{*}{\Rightarrow} v' \in a^*$ holds. Since H is an EOL system, it must be that $v \overset{+}{\Rightarrow} v' \in a^*$ holds for all $v \in K$.

In particular $a^3 \overset{+}{\Rightarrow} a^k$, for some $k \in \{0, 3\} \cup \{2^n: n \geqq 0\}$.

(1) Assume that $a^3 \overset{+}{\Rightarrow} \Lambda$. Hence $a \overset{+}{\Rightarrow} \Lambda$. Then for each $\alpha \in V$ such that $\alpha \overset{+}{\Rightarrow} x \in a^+$ it holds that $\alpha \overset{\leq t}{\Rightarrow} \Lambda$ where t equals the cardinality of V. Choose r such that $2^r > \max(\{j: \alpha \overset{\leq t}{\Rightarrow} a^j, \alpha \in V\} \cup \{0, 3\})$. Thus $a^{2^{r+1}} \in L(H)$ and by the choice of r we may write $\omega \overset{*}{\Rightarrow} x_1 \alpha x_2 \overset{t}{\Rightarrow} y_1 z y_2 = a^{2^{r+1}}$ such that $\alpha \in V, x_1 x_2 \in V^+, y_1 y_2 \in a^+$, $\alpha \overset{t}{\Rightarrow} z$ and $1 \leq |z| < 2^n$. On the other hand we have $\omega \overset{*}{\Rightarrow} x_1 \alpha x_2 \overset{t}{\Rightarrow} y_1 y_2 \in a^+$ and $2^{r+1} - 2^r = 2^r < |y_1 y_2| < 2^{r+1}$; a contradiction.

(2) Assume that $a^3 \overset{+}{\Rightarrow} a^3$. Hence there exists a t such that $a \overset{t}{\Rightarrow} a$. Consider the t^{th} speed up $\bar{H}$ of H, $L(\bar{H}) = L(H)$. (See, e.g., [7]). Hence $\bar{H}$ must have a production $a \to a$. This implies $L(\bar{H}) \in \mathcal{L}(\text{CF})$ (see. e.g., [7]); a contradiction.

(3) Assume that $a_3 \overset{+}{\Rightarrow} a^{2^n}$. If $n \leq 1$, then $a \overset{+}{\Rightarrow} \Lambda$ which yields a contradiction as in (1). Hence $n \geq 2$. This implies that $a \overset{+}{\Rightarrow} a^i$ for some $i > 1$. Hence $a^3 \overset{+}{\Rightarrow} a^{3i} \notin K \cup \{\Lambda\}$; a contradiction. □

It follows from the above that there are EOL languages that are not nbEOL languages. However the following theorem demonstrates that there is only a "small difference" between nbEOL and EOL languages.

Theorem III.1. Let $K \in \mathcal{L}(\text{EOL})$ and let § be a symbol, $\S \notin \underline{\text{alph}}\, K$. Then $K \cup \S^+ \in \mathcal{L}(\text{nbEPOL})$.

Proof. Let K and § be as in the statement of the theorem. Let $G = (V, \Sigma, P, S)$ be an sEPOL system such that $\S \notin V$, $S \in V \setminus \Sigma$ and $L(G) = K$. Moreover assume without loss of generality that N is the synchronization symbol of $G, \alpha \underset{P}{\to} N$ for each $\alpha \in V$, and $\alpha \to N$ is the only α-production for $\alpha \in \Sigma \cup \{N\}$. Then let $\bar{G} = (\bar{V}, \bar{\Sigma}, \bar{P}, S)$ be the EPOL system which is defined as follows.

(i) $W = \{[p]: p \in P\}$, $W \cap (V \cup \{\S\}) = \emptyset$, and $\bar{V} = V \cup W \cup \{\S\}$.

(ii) $\bar{\Sigma} = \Sigma \cup \{\S\}$.

(iii) $\bar{P} = \{\alpha \to [p]: p = \alpha \underset{P}{\to} x\} \cup \{[p] \to x: p = \alpha \underset{P}{\to} x\} \cup \{\alpha \to \S: \alpha \in V\} \cup \{\S \to N, \S \to NN\}$.

(1) We first show that $L(\bar{G}) = K \cup \S^+$. Let $x \in L(\bar{G})$ and let $D: S \underset{\bar{G}}{\Rightarrow} x_1 \underset{\bar{G}}{\Rightarrow} x_2 \underset{\bar{G}}{\Rightarrow} \ldots \underset{\bar{G}}{\Rightarrow} x_n = x \in \bar{\Sigma}^+$ be a derivation in $\bar{G}$. If $x \in \Sigma^+$, then clearly n is even and all productions used in D belong to $\{\alpha \to [p]: p = \alpha \underset{P}{\to} x\} \cup \{[p] \to x: p = \alpha \underset{P}{\to} x\}$. Hence $D': S \underset{G}{\Rightarrow} x_2 \underset{G}{\Rightarrow} x_4 \underset{G}{\Rightarrow} \ldots \underset{G}{\Rightarrow} x_n = x$ is a derivation in G and thus $x \in K$. If $\S \in \underline{\text{alph}}\, x$, n must be odd and consequently (the form of $\bar{P}$ implies that) $x \in \S^+$. Thus $L(\bar{G}) \subseteq K \cup \S^+$. Since each derivation step in G can be simulated in two steps in $\bar{G}$, $K \subseteq L(\bar{G})$. Moreover $S \underset{\bar{G}}{\Rightarrow} \S \underset{\bar{G}}{\Rightarrow} N^2 \underset{\bar{G}}{\Rightarrow} \S^2 \underset{\bar{G}}{\Rightarrow} N^3 \underset{\bar{G}}{\Rightarrow} \ldots$, yields $\S^+ \subseteq L(\bar{G})$. Thus $K \cup \S^+ \subseteq L(\bar{G})$. Hence $L(\bar{G}) = K \cup \S^+$.

(2) Next we show that $\bar{G}$ is nonblocking. Let $x \in \underline{\text{sent}}\, G$. A close inspection of $\bar{P}$ yields that either $x \in V^+$ or $x \in (W \cup \{\S\})^+$. If $x \in (W \cup \{\S\})^+$ then $x \underset{\bar{G}}{\Rightarrow} y \in V^+$.

If $x \in V^+$ and $|x|=k$ then $x \underset{\bar{G}}{\Rightarrow} \S^k$. Thus $x \overset{\leq 2}{\underset{\bar{G}}{\Rightarrow}} z \in \S^+$ for all $x \in \underline{\text{sent}}\, \bar{G}$. Hence $\bar{G}$ is nonblocking. □

We now turn to the comparison of the language families $\mathscr{L}(\text{E}X\text{OL})$, $\mathscr{L}(\text{nbE}X\text{OL})$, $\mathscr{L}(X\text{OL})$ where X denotes either P, D, PD or the empty word. We need the following lemmas.

Lemma III.2. (i) $\mathscr{L}(\text{EDOL}) \subseteq \mathscr{L}(\text{nbEOL})$, and
(ii) $\mathscr{L}(\text{EPDOL}) \subseteq \mathscr{L}(\text{nbEPOL})$.

Proof. (i) Our first observation is that every EDOL system generating an infinite language can be considered as an nbEOL system. Every finite non-empty language K with $\underline{\text{alph}}\, K = \Sigma$ can be generated by a nbEOL system, namely $G = (\{S\} \cup \Sigma, \Sigma, \{S \to x : x \in K\} \cup \{\alpha \to \alpha : \alpha \in \Sigma\}, S)$.
The two observations from the above conclude the proof of (i).
(ii) Analogous to (i). □

Lemma III.3. $\mathscr{L}(\text{DOL}) \setminus \mathscr{L}(\text{nbEPOL}) \neq \emptyset$.

Proof. We will prove that $K = \{ab\} \cup \{a^{2^n}bc : n \geq 1\} \in \mathscr{L}(\text{DOL}) \setminus \mathscr{L}(\text{nbEPOL})$.
(i) Let G be the DOL system which is defined by $G = (\{a, b, c\}, \{a, b, c\}, \{a \to a^2, b \to bc, c \to \Lambda\}, ab)$. Obviously $L(G) = K$. Thus $K \in \mathscr{L}(\text{DOL})$.
(ii) The fact that $K \notin \mathscr{L}(\text{nbEPOL})$ is proved by a contradiction. Assume that $K \in \mathscr{L}(\text{nbEPOL})$. Then $K = L(H)$ for an nbEPOL system $H = (V, \Sigma, P, \omega)$. Since H is nonblocking, for each $v \in K$, $v \overset{+}{\underset{H}{\Rightarrow}} v' \in K$. Thus $a^2bc \overset{t}{\underset{H}{\Rightarrow}} x \in K$ for a positive integer t. Since H is propagating, $|x| \geq 4$. Moreover x cannot equal a^2bc because this would imply that K is context-free. Thus $a^2bc \overset{t}{\underset{H}{\Rightarrow}} a^{2^n}bc$ for an $n \geq 2$. Clearly $a \overset{t}{\underset{H}{\Rightarrow}} y$ implies $y \in a^+$, thus $a \overset{t}{\underset{H}{\Rightarrow}} a^i$ for an $i > 0$. $b \overset{t}{\underset{H}{\Rightarrow}} a^k b$ ($b \overset{t}{\underset{H}{\Rightarrow}} a^k$ respectively), $k > 0$ is impossible because then $ab \overset{t}{\underset{H}{\Rightarrow}} a^{i+k}b$ ($ab \overset{t}{\underset{H}{\Rightarrow}} a^{i+k}$ respectively) which contradicts the fact that $L(H) = K$. Hence we must have $a \overset{t}{\underset{H}{\Rightarrow}} a^i$, $i > 1$ and $b \overset{t}{\underset{H}{\Rightarrow}} b$. But then $ab \overset{t}{\underset{H}{\Rightarrow}} a^i b$ which again contradicts the fact that $L(H) = K$. Thus $K \notin \mathscr{L}(\text{nbEPOL})$.
Then (i) and (ii) yield the lemma. □

Lemma III.4. $\mathscr{L}(\text{POL}) \setminus \mathscr{L}(\text{EDOL}) \neq \emptyset$.

Proof. Let $K = \{a^n : n \geq 1\}$. It is proved in [6] that $K \in \mathscr{L}(\text{POL}) \setminus \mathscr{L}(\text{EDOL})$. □

Lemma III.5. $\mathscr{L}(\text{EPDOL}) \setminus \mathscr{L}(\text{nbEDOL}) \neq \emptyset$.

Proof. We will prove that $K = \{a^2b^2, b^4(ac)^2\} \in \mathscr{L}(\text{EPDOL}) \setminus \mathscr{L}(\text{nbEDOL})$.
(i) Let G be the EPDOL system which is defined by $G = (\{A, a, b, c\}, \{a, b, c\}, \{A \to A, a \to b^2, b \to ac, c \to A\}, a^2b^2)$. Obviously $L(G) = K$. Thus $K \in \mathscr{L}(\text{EPDOL})$.

(ii) The fact that $K \notin \mathscr{L}$(nbEDOL) is proved by a contradiction. Assume that $K \in \mathscr{L}$(nbEDOL). Then $K = L(H)$ for an nbEDOL system $H = (V, \Sigma, P, \omega)$. Since H is deterministic there exists a positive integer t such that either $a^2b^2 \overset{t}{\underset{H}{\Rightarrow}} b^4 \cdot (ac)^2$ or $b^4(ac)^2 \overset{t}{\underset{H}{\Rightarrow}} a^2b^2$. The latter implies $b \overset{t}{\underset{H}{\Rightarrow}} \Lambda$ and $(ac)^2 \overset{t}{\underset{H}{\Rightarrow}} a^2b^2$ which is clearly impossible. Hence $a^2b^2 \overset{t}{\underset{H}{\Rightarrow}} b^4(ac)^2$. There are three cases to consider.

(a) $a \overset{t}{\underset{H}{\Rightarrow}} \Lambda$. Then however $b^2 \overset{t}{\underset{H}{\Rightarrow}} b^4(ac)^2$ which contradicts the fact that H is deterministic.

(b) $a \overset{t}{\underset{H}{\Rightarrow}} b$. Then however $b^2 \overset{t}{\underset{H}{\Rightarrow}} b^2(ac)^2$ which contradicts the fact that H is deterministic.

(c) $a \overset{t}{\underset{H}{\Rightarrow}} b^2$. Then $b^2 \overset{t}{\underset{H}{\Rightarrow}} (ac)^2$. The fact that H is deterministic yields $b \overset{t}{\underset{H}{\Rightarrow}} ac$. Observe that

(III.1)... $a \overset{*}{\underset{H}{\Rightarrow}} x$ implies $|x| \geqq 1$, and $b \overset{*}{\underset{H}{\Rightarrow}} x$ implies $|x| \geqq 1$. Clearly $a^2b^2 \overset{t}{\underset{H}{\Rightarrow}} b^4 \cdot (ac)^2 \overset{t}{\underset{H}{\Rightarrow}} (ac)^4(b^2x_1)^2 \overset{t}{\underset{H}{\Rightarrow}} (b^2x_1)^4((ac)^2x_2)^2 = z$ for some $x_1, x_2 \in V^*$.

Now the form of z and (III.1) yield that

(III.2)... for all words v such that $z \overset{*}{\underset{H}{\Rightarrow}} v$, $|v| \geqq 12$. Since the longest word of $L(H) = K$ has length 8, (III.2) contradicts the fact that H is nonblocking. Having established a contradiction for all possible cases, we get that $K \notin \mathscr{L}$(nbEDOL) which concludes the proof of (ii).

Hence the lemma holds. □

Lemma III.6. $\mathscr{L}$(nbEPDOL)$\setminus\mathscr{L}$(OL)$\neq\emptyset$.

Proof. We will prove that $K = \{a^{2^{5n}}b : n \geqq 0\} \cup \{a^{2^{5n+2}}c : n \geqq 0\} \in \mathscr{L}$(nbEPDOL)$\setminus$ $\setminus\mathscr{L}$(OL).

(i) Let G be the nbEPDOL system which is defined by $G = (\{A, B, C, a, b, c\}, \{a, b, c\}, \{A \to c, B \to C, C \to b, a \to aa, b \to A, c \to B\}, ab)$. Obviously $L(G) = K$. Thus $K \in \mathscr{L}$(nbEPDOL).

(ii) The fact that $K \notin \mathscr{L}$(OL) is proved by a contradiction. Assume that $K \in$ $\in \mathscr{L}$(OL). Then $K = L(H)$ for a OL system $H = (V, V, P, \omega)$. Without loss of generality we can assume that $V = \{a, b, c\}$.

(ii.1) Clearly $a \underset{P}{\to} x$ implies $x \in a^*$, $b \underset{P}{\to} x$ implies $x \in a^*b \cup a^*c$; and $c \underset{P}{\to} x$ implies $x \in a^*b \cup a^*c$ (otherwise $L(H)$ would contain words not belonging to K).

(ii.2) The set P contains only one a-production. For assume to the contrary that there exist two different a-productions in P, say $a \to a^i$ and $a \to a^j$, $i > j$. Let $b \to x$ be an arbitrary b-production of P. Then for all $n \geqq 0$, $a^{2^{5n}}b \underset{H}{\Rightarrow} a^{2^{5n} \cdot i}x$ and

$a^{25n}b \underset{H}{\Rightarrow} a^{25n\cdot i-i+j}x$. Thus for all $n \geqq 0$, $a^{25n\cdot i}x$ and $a^{25n\cdot i-i+j}x$ belong to $L(H)$ which (for n large enough) contradicts the fact that $L(H)=K$.

(ii.3) The only a-production of P cannot be $a \to \Lambda$ otherwise $L(H)$ would be finite, a contradiction.

(ii.4) Analogously to (ii.2) we can prove that P contains only one b-production and one c-production.

Now (ii.1) through (ii.4) yield that H must be a PDOL system.

Hence $ab \underset{H}{\Rightarrow} a^4c \underset{H}{\Rightarrow} a^{32}b$. There are four cases to consider.

(a) $a \underset{H}{\Rightarrow} a$ and $b \underset{H}{\Rightarrow} a^3c$. Then however $a^{32}b \underset{H}{\Rightarrow} a^{35}c$; a contradiction.

(b) $a \underset{H}{\Rightarrow} a^2$ and $b \underset{H}{\Rightarrow} a^2c$. Then however $a^{32}b \underset{H}{\Rightarrow} a^{66}c$; a contradiction.

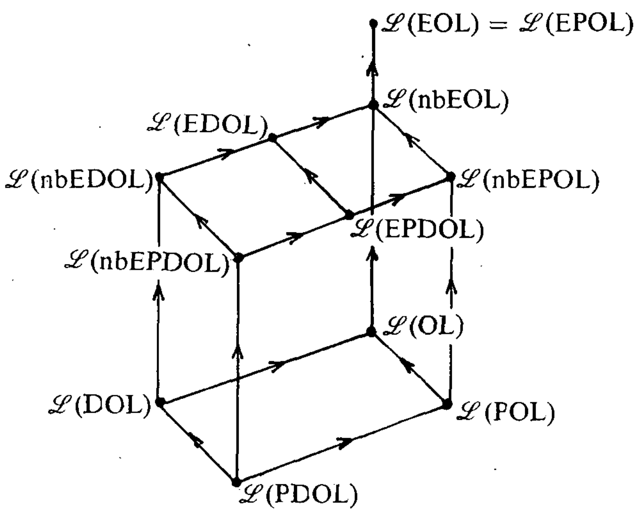
(c) $a \underset{H}{\Rightarrow} a^3$ and $b \underset{H}{\Rightarrow} ac$. Then however $a^{32}b \underset{H}{\Rightarrow} a^{97}c$; a contradiction.

(d) $a \underset{H}{\Rightarrow} a^4$ and $b \underset{H}{\Rightarrow} c$. Then $a^4c \underset{H}{\Rightarrow} a^{32}b$, $a \underset{H}{\Rightarrow} a^4$ and the fact that H is deterministic yield $c \underset{H}{\Rightarrow} a^{16}b$. Then however $a^{128}c \underset{H}{\Rightarrow} a^{528}b$; a contradiction.

Having established a contradiction for all possible cases, we get $K \notin \mathscr{L}(\mathrm{OL})$. Then (i) and (ii) yield the lemma. □

We are now ready to state the main result of the section. As expected, if X denotes either P, D, PD or the empty word, we have that $\mathscr{L}(X\mathrm{OL}) \subset \mathscr{L}(\mathrm{nbE}X\mathrm{OL}) \subset \mathscr{L}(\mathrm{E}X\mathrm{OL})$.

Theorem III.2. The following diagram holds:



where, if there is a directed chain of edges in the diagram leading from a class X to a class Y then $X \subset Y$; otherwise X and Y are incomparable but not disjoint.

Proof. It is well known that $\mathscr{L}(\mathrm{EOL}) = \mathscr{L}(\mathrm{EPOL})$ (see, e.g., [7]). Inclusions follow from the definitions and Lemma III.2; strict inclusions and incomparabilities follow from Lemma III.1 and Lemmas III.3 through III.6. □

IV. Systems with tables

In the case of E(P)TOL systems the nonblocking restriction turns out to be no restriction with respect to the language generating power. This contrasts the results of the previous section.

Theorem IV.1. $\mathscr{L}$(nbEPTOL) $=\mathscr{L}$(nbETOL) $=\mathscr{L}$(EPTOL) $=\mathscr{L}$(ETOL).

Proof. We shall show that $\mathscr{L}(\text{ETOL})\subseteq\mathscr{L}(\text{nbEPTOL})$. The theorem then follows from the definitions. Let $K\in\mathscr{L}(\text{ETOL})$. Then (see [6]) there exists a PTOL system $G=(V, V, \{P_1, P_2, \dots, P_k\}, \omega)$, $k\geqq 1$ and a Λ-free homomorphism h: $V^*\to\Sigma^*$, such that $h(L(G))=K$. Without loss of generality assume that $V\cap\Sigma=\emptyset$. For $1\leqq i\leqq k$ let $Q_i=P_i\cup\{\alpha\to\alpha:\alpha\in\Sigma\}$. Let $Q=\{\alpha\to h(\alpha):\alpha\in V\}\cup\{\alpha\to\alpha:\alpha\in\Sigma\}$. Finally define the EPTOL system $\bar{G}$ by $\bar{G}=(V\cup\Sigma, \Sigma, \{Q_1, Q_2, \dots, Q_k, Q\}, \omega)$. Clearly $\bar{G}$ is nonblocking and $L(\bar{G})=K$. Thus $K\in\mathscr{L}(\text{nbEPTOL})$. Hence $\mathscr{L}(\text{ETOL})\subseteq\mathscr{L}(\text{nbEPTOL})$. □

Even in the case of E(P)DTOL systems the nonblocking condition has no consequences for the generating power of those systems. We first prove the following lemma.

Lemma IV.1. $\mathscr{L}(\text{EPDTOL})\subseteq\mathscr{L}(\text{nbEPDTOL})$.

Proof. Let $G=(V, \Sigma, \mathscr{P}, S)$ be an EPDTOL system where $\mathscr{P}=\{P_1, P_2, \dots, P_k\}$, $k\geqq 1$. Without loss of generality assume that $S\in V\setminus\Sigma$, $L(G)\neq\emptyset$ and $\underline{\text{alph}}\, L(G)=\Sigma$. Let $\bar{V}=\{\bar{\alpha}:\alpha\in V\}$, $\bar{V}\cap V=\emptyset$ and let $\bar{h}$ be the homomorphism on $\bar{V}^*$ defined by $\bar{h}(\alpha)=\bar{\alpha}$ for $\alpha\in V$. For each $\emptyset\neq X\subseteq V$ let w_X be a fixed word such that $\underline{\text{alph}}\, w_X=X$ and each letter occurs precisely once in w_X. Furthermore let $G_X=(V', \bar{\Sigma}, \bar{\mathscr{P}}', \bar{h}(w_X))$ be the ETOL system which is defined as follows. $V'=V\cup\bar{V}$, and $\mathscr{P}'=\{P': P\in\mathscr{P}\}$ where for $P\in\mathscr{P}$, $P'=P\cup\{\bar{h}(\alpha)\to x: \alpha\underset{P}{\to}x\}$. Then $\text{SUC}(G)=\{\emptyset\neq X\subseteq V: L(G_X)\neq\emptyset\}$, in other words for a $w\in V^+$, $\underline{\text{alph}}\, w\in\text{SUC}(G)$ if and only if there exists a $w'\in\Sigma^+$ such that $w\overset{+}{\underset{G}{\Rightarrow}}w'$. For $X\in\text{SUC}(G)$ we define $\underline{\text{next}}\, X=\{i: P_i\in\mathscr{P}, w_X\underset{P_i}{\Rightarrow}y, \underline{\text{alph}}\, y\in\text{SUC}(G) \text{ or } \underline{\text{alph}}\, y\subseteq\Sigma\}$. Now we will construct an nbEPDTOL system H such that $L(G)=L(H)$. We proceed as follows. $\hat{V}=\{S\}\cup\Sigma\cup\bigcup_{X\in\text{SUC}(G)}\{[\alpha, X]_i: \alpha\in\underline{\text{alph}}\, X, i\in\underline{\text{next}}\, X\}$, $\hat{V}\cap(V\setminus(\{S\}\cup\Sigma))=\emptyset$. For $i\in\underline{\text{next}}\,\{S\}$, define $Q_{\text{in},i}=\{S\to[S, \{S\}]_i\}\cup\{\alpha\to\alpha: \alpha\in\hat{V}\setminus\{S\}\}$. For $X\in\text{SUC}(G)$, $w_X\underset{P_i}{\Rightarrow}y$, $\underline{\text{alph}}\, y=Y\in\text{SUC}(G)$ and $j\in\underline{\text{next}}\, Y$ define

$$Q_{X,i,j}=\{[\alpha, X]_i\to[\beta_1, Y]_j[\beta_2, Y]_j\dots[\beta_m, Y]_j: \alpha\in X, \alpha\underset{P_i}{\to}\beta_1\beta_2\dots\beta_m, m\geqq 1, \beta_l\in V$$
$$\text{for } 1\leqq l\leqq m\}\cup\{\alpha\to\alpha: \alpha\in\hat{V}\setminus\{[\beta, X]_i: \beta\in X\}\}.$$

For $X\in\text{SUC}(G)$, $w_X\underset{P_i}{\Rightarrow}y$, $\underline{\text{alph}}\, y\subseteq\Sigma$ define

$$Q_{X,i,\text{fin}}=\{[\alpha, X]_i\to z: \alpha\in X, \alpha\underset{P_i}{\to}z\}\cup\{\alpha\to\alpha: \alpha\in\hat{V}\setminus\{[\beta, X]_i: \beta\in X\}\}.$$

Let $\hat{P}=\{Q_{\text{in},i}: i\in\underline{\text{next}}\ \{S\}\}\cup\{Q_{X,i,j}: X\in\text{SUC}(G),\ w_X\underset{P_i}{\Rightarrow}y,\ \underline{\text{alph}}\ y=Y\in\text{SUC}(G)$ and $j\in\underline{\text{next}}\ y\}\cup\{Q_{X,i,\text{fin}}: X\in\text{SUC}(G),\ w_X\underset{P_i}{\Rightarrow}y,\ \underline{\text{alph}}\ Y\subseteq\Sigma\}$.

Finally let H be the EPDTOL system defined by $H=(\hat{V},\Sigma,\hat{\mathscr{P}},S)$. First we show that $L(H)=L(G)$. For $X\in\text{SUC}(G)$ and $i\in\underline{\text{next}}\ X$ the homomorphism $h_{X,i}$ on V^* is defined by $h_{X,i}(\alpha)=[\alpha,X]_i$ if $\alpha\in V$; furthermore the homomorphism g on $\hat{V}^*$ is defined by $g(\alpha)=\alpha$ if $\alpha\in\{S\}\cup\Sigma$ and $g([\alpha,X]_i)=\alpha$ if $X\in\text{SUC}(G)$, $\alpha\in X$ and $i\in\underline{\text{next}}\ X$. Let $x\in L(G)$, thus $S=x_0\underset{P_{i_1}}{\Rightarrow}x_1\underset{P_{i_2}}{\Rightarrow}x_2\Rightarrow\dots\underset{P_{i_n}}{\Rightarrow}x_n=x$, $n\geqq 1$, $i_1,\dots,i_n\in\{1,\dots,k\}$. Then obviously, if for $0\leqq l\leqq n$ we denote $\underline{\text{alph}}\ x_l=X_l$,

$$S\underset{Q_{in,i_1}}{\Rightarrow}h_{X_0,i_1}(x_0)\underset{Q_{X_0,i_1,i_2}}{\Rightarrow}h_{X_1,i_2}(x_1)\Rightarrow\dots\underset{Q_{X_{n-2},i_{n-1},i_n}}{\Rightarrow}h_{X_{n-1},i_n}(x_{n-1})\underset{Q_{X_{n-1},i_n,\text{fin}}}{\Rightarrow}x_n=x.$$

Consequently $x\in L(H)$. Hence $L(G)\subseteq L(H)$.

Conversely let $x\in L(H)$ and let D: $S=x_0\underset{H}{\Rightarrow}x_1\underset{H}{\Rightarrow}x_2\underset{H}{\Rightarrow}\dots\underset{H}{\Rightarrow}x_n=x$ be a shortest derivation of x in H. Thus, if for $0\leqq l\leqq n$ we denote $\underline{\text{alph}}\ g(x_l)=X_l$,

$$D\colon S=x_0\underset{Q_{\text{in},i_1}}{\Rightarrow}x_1\underset{Q_{X_1,i_1,i_2}}{\Rightarrow}x_2\Rightarrow\dots\underset{Q_{X_{n-2},i_{n-2},i_{n-1}}}{\Rightarrow}x_{n-1}\underset{Q_{X_{n-1},i_{n-1},\text{fin}}}{\Rightarrow}x_n=x,$$

$n\geqq 2$ and $i_1,\dots,i_{n-1}\in\{1,\dots,k\}$. Consequently

$$S=x_0\underset{P_{i_1}}{\Rightarrow}g(x_2)\underset{P_{i_2}}{\Rightarrow}\dots\underset{P_{i_{n-2}}}{\Rightarrow}g(x_{n-1})\underset{P_{i_{n-1}}}{\Rightarrow}g(x_n)=x$$

and thus $x\in L(G)$. Hence $L(H)\subseteq L(G)$.

We end the proof of the lemma by showing that H is nonblocking. Let $x\in\underline{\text{sent}}\ H$. Then there are three possible cases: $x=S$ or $x\in\Sigma^+$ or $x=h_{X,i}(v)$, $v\in V^+$, $X\in\text{SUC}(G)$ and $i\in\underline{\text{next}}\ X$. Since $L(H)=L(G)\neq\emptyset$ it suffices to consider sentential forms of the third kind. Thus $x=h_{X,i}(v)$, $v\in V^+$, $X\in\text{SUC}(G)$ and $i\in\underline{\text{next}}\ X$. Hence there exist v' and v'' such that $v\underset{P_i}{\Rightarrow}v'\overset{*}{\underset{G}{\Rightarrow}}v''\in\Sigma^+$. Then inspecting the proof of $L(G)\subseteq L(H)$ one can easily see that $x=h_{X,i}(v)\overset{*}{\underset{H}{\Rightarrow}}v''$ which shows that H is nonblocking. □

As a corollary we obtain the answer to an open problem stated in [6].

Definition IV.1. A language L is contained in $\mathscr{L}$(NPDTOL) if and only if there exists a PDTOL system H and a non-erasing homomorphism h such that $L=h(L(H))$.

Corollary IV.1. $\mathscr{L}$(NPDTOL)$=\mathscr{L}$(EPDTOL).

Proof. We will use the notation from the proof of Lemma IV.1. Fix a $u_s\in\Sigma^+$ such that $S\overset{+}{\underset{G}{\Rightarrow}}u_s$ and for each $X\in\text{SUC}(G)$ let D_X: $w_X\overset{+}{\underset{G}{\Rightarrow}}u_X\in\Sigma^+$ be a fixed derivation. Then define the Λ-free homomorphism h on $\hat{V}^*$ as follows: $h(S)=u_S$, $h([\alpha,X]_i)=\underline{\text{ctr}}_{D_X,w_X}\alpha$ if $X\in\text{SUC}(G)$, $\alpha\in\underline{\text{alph}}\ X$ and $i\in\underline{\text{next}}\ X$, and $h(\alpha)=\alpha$ if

$\alpha \in \Sigma$. Let $\hat{H}'$ be the PDTOL system defined by $\hat{H}' = (\hat{V}, \hat{V}, \hat{\mathscr{P}}, S)$. Clearly $L(G) = h(L(\hat{H}'))$. Hence $\mathscr{L}(\text{EPDTOL}) \subseteqq \mathscr{L}(\text{NPDTOL})$. Since also $\mathscr{L}(\text{NPDTOL}) \subseteqq \mathscr{L}(\text{EPDTOL})$ (see [6]), the corollary holds. □

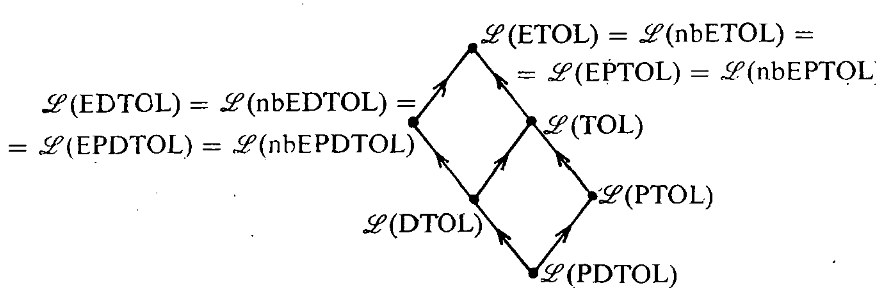
For the deterministic case we obtain a result analogous to the statement of Theorem IV.1.

Theorem IV.2. $\mathscr{L}(\text{nbEPDTOL}) = \mathscr{L}(\text{nbEDTOL}) = \mathscr{L}(\text{EPDTOL}) = \mathscr{L}(\text{EDTOL})$.

Proof. From the definitions we get $\mathscr{L}(\text{nbEPDTOL}) \subseteqq \mathscr{L}(\text{nbEDTOL}) \subseteqq \mathscr{L}(\text{EDTOL})$. It is well known (see, e.g., [1]) that $\mathscr{L}(\text{EDTOL}) = \mathscr{L}(\text{EPDTOL})$. From Lemma IV.1 we get $\mathscr{L}(\text{EPDTOL}) \subseteqq \mathscr{L}(\text{nbEPDTOL})$. Combining the above results, the theorem immediately follows. □

Let X and Y denote P, D, PD or the empty word. Then Theorem IV.1 and Theorem IV.2 show that $\mathscr{L}(\text{nbE}X\text{TOL}) = \mathscr{L}(\text{E}X\text{TOL})$. Thus comparing $\mathscr{L}(\text{nbE}X\text{TOL})$ and $\mathscr{L}(Y\text{TOL})$ is the same as comparing $\mathscr{L}(\text{E}X\text{TOL})$ and $\mathscr{L}(Y\text{TOL})$. For completeness only we present here the diagram in the case of tabled L systems. The proof is given using well known results from the literature.

Theorem IV.3. The following diagram holds:



where, if there is a directed chain of edges in the diagram leading from a class X to a class Y then $X \subset Y$; otherwise X and Y are incomparable but not disjoint.

Proof. Inclusions follow from the definitions, equalities follow from Theorem IV.1 and Theorem IV.2. Strict inclusions and incomparabilities follow from the following three observations.

(i) $\{ba^{2^n} : n \geqq 0\} \cup \{bc^{3^n} : n \geqq 0\} \in \mathscr{L}(\text{DTOL}) \setminus \mathscr{L}(\text{PTOL})$ (see, e.g., [3]).

(ii) $\{w \in \{a, b\}^* : |w| = 2^n \text{ for some } n \geqq 0\} \in \mathscr{L}(\text{PTOL}) \setminus \mathscr{L}(\text{EDTOL})$ (see, e.g., [7]).

(iii) All finite languages are in $\mathscr{L}(\text{EDTOL})$ and there are finite languages which are not TOL languages (see, e.g., [3]). □

Since emptiness is a decidable property for ETOL systems (see, e.g., [7]) and since all constructions used in this section are effective, it follows that for every system, considered in this section, generating a non-empty language, there exists effectively an equivalent nonblocking system. This contrasts Theorem II.4. Moreover it turns out that nonblocking is a decidable property for ETOL systems. This result should be compared with Theorem II.5.

Theorem IV.4. Let G be an ETOL system. Then it is decidable whether or not G is nonblocking.

Proof. Let $G=(V, \Sigma, \mathscr{P}, \omega)$ be an ETOL system. Let $\bar{V}=\{\bar{\alpha}: \alpha \in V\}$, $\bar{V} \cap V=\emptyset$ and let $\bar{h}$ be the homomorphism on V^* defined by $\bar{h}(\alpha)=\bar{\alpha}$ for $\alpha \in V$. For each $\emptyset \neq X \subseteq V$ let w_X be a fixed word such that $\mathrm{alph}\, w_X=X$ and each letter occurs precisely once in w_X. Furthermore let $G_X=(V', \Sigma, \mathscr{P}', \bar{h}(w_X))$ be the ETOL system which is defined as follows. $V'=V \cup \bar{V}$, and $\mathscr{P}'=\{P': P \in \mathscr{P}\}$ where for $P \in \mathscr{P}$, $P'=P \cup \{\bar{h}(\alpha) \to x: \alpha \xrightarrow[P]{} x\}$. Let $\mathscr{A}=\{G_X: \mathrm{sent}\, G \cap \{x \in X^*: \mathrm{alph}\, x=X\} \neq \emptyset\}$. Obviously G is nonblocking if and only if $\mathscr{A} \neq \emptyset$ and for each $H \in \mathscr{A}$, $L(H) \neq \emptyset$. The decidability of the latter question follows from the closure properties of $\mathscr{L}$(ETOL), the effectiveness of the construction of $\mathscr{A}$ and the decidability of the emptiness problem for ETOL systems. Hence the theorem holds. □

Discussion

In this paper we have investigated the effect that the nonblocking restriction has on the language generating power of various classes of rewriting systems. Since the blocking facility forms a typical "programming tool" in generating a language, we believe that our results shed some light on the nature of the generation of languages by grammars.

The research started in this paper can be continued in several directions.

(1) The class of languages generated by the "nonblocking subclass" of a class X of rewriting systems should be often investigated on its own (whenever the nonblocking restriction influences the language generating power of the class X). Such a typical candidate to investigate is $\mathscr{L}$(nbEOL); for example the closure properties and the combinatorial properties of languages in this class. Also the decidability status of the question "Does an arbitrary EOL system generate a language in $\mathscr{L}$(nbEOL)?" forms an interesting open problem.

(2) The role of the nonblocking restriction in classes of rewriting systems different from those investigated in this paper should also be investigated.

(3) Clearly the way that we have formally defined the nonblocking of a rewriting system is only one of several possibilities. Other possibilities should also be investigated.

(4) A nonblocking condition can be also defined for various types of automata, for example one could require that for every state of an automaton there exists a computation that leads from this state to an accepting state. (Conditions of this type are often considered in the theory of Petri-nets (see, e.g., [2]), where they are referred to as "liveness conditions".) The effect of nonblocking on the generative power of various classes of automata should be investigated.

Appendix

Here we give the full proof of Lemma II.2.

For every context-sensitive grammar, generating a non-empty language there exists an equivalent nonblocking context-sensitive grammar.

Proof. Let $K \subseteq \Sigma^*$ be a non-empty language generated by a context-sensitive grammar.

1) If K is finite, then let $G=(\Sigma \cup \{S\}, \Sigma, P, S)$ be the context-sensitive grammar with $P=\{S \to x: x \in K\}$. Obviously; G is a nonblocking context-sensitive grammar and $L(G)=K$.

2) If K is infinite, we proceed as follows. Let $\Sigma'=\{[a, b, c, d]: a, b, c, d \in \Sigma\} \cup \cup \{[a, b, c]: a, b, c \in \Sigma\} \cup \{[a, b]: a, b \in \Sigma\} \cup \{[a]: a \in \Sigma\}$ with $\Sigma' \cap \Sigma = \emptyset$.

Let $K'=\{[a_1, a_2, a_3, a_4] \dots [a_{4n-3}, a_{4n-2}, a_{4n-1}, a_{4n}]: n \geqq 2, a_i \in \Sigma$, for $1 \leqq i \leqq 4n$, and $a_1 a_2 \dots a_{4n} \in K\} \cup \{[a_1, a_2, a_3, a_4] \dots [a_{4n-3}, a_{4n-2}, a_{4n-1}, a_{4n}][a_{4n+1}]: n \geqq 2, a_i \in \Sigma$, for $1 \leqq i \leqq 4n+1$, and $a_1 a_2 \dots a_{4n+1} \in K\} \cup \{[a_1, a_2, a_3, a_4] \dots [a_{4n-3}, a_{4n-2}, a_{4n-1}, a_{4n}][a_{4n+1}, a_{4n+2}]: n \geqq 2, a_i \in \Sigma$, for $1 \leqq i \leqq 4n+2$, and $a_1 a_2 \dots a_{4n+2} \in K\} \cup \{[a_1, a_2, a_3, a_4] \dots [a_{4n-3}, a_{4n-2}, a_{4n-1}, a_{4n}][a_{4n+1}, a_{4n+2}, a_{4n+3}]: n \geqq 2, a_i \in \Sigma$, for $1 \leqq i \leqq 4n+3$, and $a_1 a_2 \dots a_{4n+3} \in K\}$.

Let h be the homomorphism from Σ'^* into Σ^* defined by $h([a_1, a_2, a_3, a_4]) = a_1 a_2 a_3 a_4$, $h([a_1, a_2, a_3]) = a_1 a_2 a_3$, $h([a_1, a_2]) = a_1 a_2$ and $h([a_1]) = a_1$, for $a_i \in \Sigma$, $1 \leqq i \leqq 4$. Clearly $h(K') = K \setminus \{x \in K: |x| < 8\}$ and hence $K' \in \mathscr{L}(\mathrm{CS})$. (See, e.g., [4].) Let $G'=(V', \Sigma', P', S')$ be a context-sensitive grammar, such that $(V' \setminus \Sigma') \cap \cap \Sigma = \emptyset$ and $L(G')=K'$. Without loss of generality we assume that no terminals occur in the left-hand side of any production of P'.

The context-sensitive grammar $G=(V, \Sigma, P, S)$ is defined as follows. $V = \bar{V} \cup V' \cup \Sigma$, where $\bar{V}=\{S, L, R, L_1, R_1, N, N_L, \vec{N}, \vec{B}, \overleftarrow{B}, \vec{M}_0, \overleftarrow{M}_0, \vec{M}_1, \overleftarrow{M}_1, M_2, \vec{M}_2, \vec{M}_3, \overleftarrow{M}_3, X_1, X_2\}$ and $\bar{V} \cap (V' \cup \Sigma) = \emptyset$.

P consists of the following productions.

(1) $S \to x$, if $x \in K$ and $|x| < 8$.

(2) $S \to L\vec{M}_0 S'R$.

(3) All productions from P'.

(4) $\vec{M}_0 \alpha \to \alpha \vec{M}_0$, if $\alpha \in \Sigma'$.

(5) $\vec{M}_0 \to \overleftarrow{B}$.

(6) $[a_1, a_2, a_3, a_4]\vec{M}_0 R \to \overleftarrow{M}_0 a_1 a_2 a_3 a_4$,
$[a_1, a_2, a_3, a_4][a_5]\vec{M}_0 R \to \overleftarrow{M}_0 a_1 a_2 a_3 a_4 a_5$,
$[a_1, a_2, a_3, a_4][a_5, a_6]\vec{M}_0 R \to \overleftarrow{M}_0 a_1 a_2 a_3 a_4 a_5 a_6$ and
$[a_1, a_2, a_3, a_4][a_5, a_6, a_7]\vec{M}_0 R \to \overleftarrow{M}_0 a_1 a_2 a_3 a_4 a_5 a_6 a_7$, for $a_i \in \Sigma$, $1 \leqq i \leqq 7$.

(7) $\alpha \overleftarrow{B} \to \overleftarrow{B} N$, if $\alpha \in V' \cup \Sigma \cup \{N, L_1, \vec{N}, M_2, \vec{M}_2\}$.

(8) $L\overleftarrow{B} \to N_L \vec{B}$ and $N_L \overleftarrow{B} \to N_L \vec{B}$.

(9) $[a_1, a_2, a_3, a_4][a_5, a_6, a_7, a_8]\overleftarrow{M}_0 \to [a_1, a_2, a_3, a_4]\overleftarrow{M}_0 a_5 a_6 a_7 a_8$ for $a_i \in \Sigma$, $1 \leqq i \leqq 8$.

(10) $L[a_1, a_2, a_3, a_4]\overleftarrow{M}_0 \to a_1 a_2 a_3 a_4$, for $a_i \in \Sigma$, $1 \leqq i \leqq 4$.

(11) $\vec{B}\alpha \to N\vec{B}$, if $\alpha \in V' \cup \{N\}$,

(12) $\vec{B}R \to NL_1\vec{M}_1S'R_1$ and $\vec{B}R_1 \to NL_1\vec{M}_1S'R_1$.
(13) $\vec{M}_1\alpha \to \alpha\vec{M}_1$, if $\alpha \in \Sigma'$.
(14) $\vec{M}_1 \to \overleftarrow{B}$.
(15) $\vec{M}_1R_1 \to \overleftarrow{M}_1R_1$.
(16) $\alpha\overleftarrow{M}_1 \to \overleftarrow{M}_1\alpha$, if $\alpha \in \Sigma'$.
(17) $L_1\overleftarrow{M}_1 \to NM_2$.
(18) $NM_2 \to M_2\vec{N}$.
(19) $N_LM_2 \to N_L\vec{M}_2$.
(20) $\vec{N}\alpha \to \alpha\vec{N}$, if $\alpha \in \Sigma$.
(21) $\vec{N}[a_1, a_2, a_3, a_4] \to a_1a_2a_3a_4$, for $a_i \in \Sigma$, $1 \leqq i \leqq 4$.
(22) $\vec{N}[a_1, a_2, a_3] \to \overleftarrow{B}N$, $\vec{N}[a_1, a_2] \to \overleftarrow{B}N$, $\vec{N}[a_1] \to \overleftarrow{B}N$ and $\vec{N}R_1 \to \overleftarrow{B}NR_1$, for $a_i \in \Sigma$, $1 \leqq i \leqq 3$.
(23) $\vec{M}_2\alpha \to \alpha\vec{M}_2$, if $\alpha \in \Sigma$.
(24) $\vec{M}_2[a_1, a_2, a_3, a_4] \to a_1a_2a_3a_4\vec{M}_3$, for $a_i \in \Sigma$, $1 \leqq i \leqq 4$.
(25) $\vec{M}_2[a_1, a_2, a_3] \to \overleftarrow{B}N$, $\vec{M}_2[a_1, a_2] \to \overleftarrow{B}N$, $\vec{M}_2[a_1] \to \overleftarrow{B}N$ and $\vec{M}_2R_1 \to \overleftarrow{B}NR_1$, for $a_i \in \Sigma$, $1 \leqq i \leqq 3$.
(26) $\vec{M}_3[a_1, a_2, a_3, a_4] \to \overleftarrow{M}_3[a_1, a_2, a_3, a_4]$, for $a_i \in \Sigma$, $1 \leqq i \leqq 4$.
(27) $\vec{M}_3[a_1, a_2, a_3] \to \overleftarrow{B}N$, $\vec{M}_3[a_1, a_2] \to \overleftarrow{B}N$, $\vec{M}_3[a_1] \to \overleftarrow{B}N$ and $\vec{M}_3R \to \overleftarrow{B}NR_1$, for $a_i \in \Sigma$, $1 \leqq i \leqq 3$.
(28) $\alpha\overleftarrow{M}_3 \to \overleftarrow{M}_3\alpha$, if $\alpha \in \Sigma$.
(29) $N_L\overleftarrow{M}_3 \to X_1X_2$.
(30) $X_1X_2\alpha \to \alpha X_1X_2$, if $\alpha \in \Sigma$.
(31) $X_1X_2[a_1, a_2, a_3, a_4][a_5, a_6, a_7, a_8] \to a_1a_2a_3a_4X_1X_2[a_5, a_6, a_7, a_8]$, for $a_i \in \Sigma$, $1 \leqq i \leqq 8$.
(32) $X_1X_2[a_1, a_2, a_3, a_4][a_5, a_6, a_7]R_1 \to a_1a_2a_3a_4a_5a_6a_7$,
$X_1X_2[a_1, a_2, a_3, a_4][a_5, a_6]R_1 \to a_1a_2a_3a_4a_5a_6$,
$X_1X_2[a_1, a_2, a_3, a_4][a_5]R_1 \to a_1a_2a_3a_4a_5$ and
$X_1X_2[a_1, a_2, a_3, a_4]R_1 \to a_1a_2a_3a_4$, for $a_i \in \Sigma$, $1 \leqq i \leqq 7$.

First we show that $L(G) \subseteq K$. Starting from the axiom S only productions from (1) and (2) can be applied, resulting either in a word $x \in K$, $|x| < 8$, or in a word of $\underline{\text{sent}}\, G$ of type A, i.e. of the form $Lx\vec{M}_0yR$, with $x \in \Sigma'^*$ and $xy \in \underline{\text{sent}}\, G'$.

The productions, applicable to words of $\underline{\text{sent}}\, G$ which are of type A belong to (3), (4), (5) and (6). If a production from (3) or (4) is applied to a word of type A, the resulting word again is of type A.

If a production from (5) is applied to a word of $\underline{\text{sent}}\, G$ of type A, we get a word of type B, i.e. of the form $Lx\overleftarrow{B}yR$, with $xy \in (V' \cup \{N\})^*$. If a production from (6) is applied to a word of type A, the resulting word is of type C, i.e. of the form $Lx\overleftarrow{M}_0y$, with $x \in \Sigma'^+$, $y \in \Sigma^+$, $h(x)y \in K$ and $|h(x)y| \geqq 8$.

The productions, applicable to words of type B come from (3), (7) or (8). Application of productions from (3) and (7) to a word of type B again yields a word of type B, whereas application of productions from (8) yields a word of type D, i.e. of the form $N_LN^*\vec{B}xR$ or $N_LN^*\vec{B}xR_1$, $x \in (V' \cup \{N\})^*$.

The productions, applicable to words of type C belong to (9) or (10). Application of a production from (9) to a word of type C yields a word of the same type, whereas application of a production from (10) yields a word of K.

The productions, applicable to words of type D belong to (3), (11) or (12). The application of a production from (3) or (11) to a word of type D results in a word of the same type; the application of a production from (12) yields a word of type E, i.e. of the form $N_L N^+ L_1 x \vec{M}_1 y R_1$, $x \in \Sigma'^*$, $xy \in \underline{\text{sent}}\ G'$.

The productions, applicable to a word of type E come from (3), (13), (14) or (15). Application of a production from (3) or (13) to a word of type E yields a word of the same type. Application of a production from (14) to a word of type E yields a word of type F, i.e. of the form $N_L x \overleftarrow{B} y R_1$ with $xy \in (V' \cup \{L_1, N\})^*$. Application of a production from (15) to a word of type E yields a word of type $\hat{G}$, i.e. of the form $N_L N^+ L_1 x \overleftarrow{M}_1 y R_1$ with $xy \in \Sigma'^+$, $h(xy) \in K$ and $|h(xy)| \geqq 8$.

The productions, applicable to a word of type F come from (3), (7) or (8), and if applied, yield words of type F, type F and type D respectively.

The productions, applicable to a word of type $\hat{G}$, belong to (16) or (17), and, if applied, yield respectively words of type $\hat{G}$ and type H, i.e. of the form $N_L N^* M_2(\{\vec{N}\} \cup \Sigma)^* \Sigma'^* R_1$, and furthermore if a word has this form, then also $h(\underline{\text{Pres}}_{\Sigma \cup \Sigma'}\, w) = w' \in K$ with $|w'| \geqq 8$.

The productions, applicable to a word of type H belong to (18), (19), (20), (21) or (22) and then yield words of type H, type I, type H, type H or type J respectively, where type I and type J are defined as follows.

A word w is of type I if $w \in N_L \Sigma^* \vec{M}_2(\{\vec{N}\} \cup \Sigma)^* \Sigma'^* R_1$ and $h(\underline{\text{Pres}}_{\Sigma \cap \Sigma'}\, w) = w' \in K$, with $|w'| \geqq 8$.

A word is of type J if it is of the form $N_L N^* M_2 x \overleftarrow{B} N^+ R_1$, with $x \in (\Sigma \cup \{N, \vec{N}\})^*$, or $N_L \Sigma^* \vec{M}_2 y B \overleftarrow{N}^+ R_1$, with $y \in (\Sigma \cup \{N, \vec{N}\})^*$, or $N_L \Sigma^* \overleftarrow{B} N^+ R_1$.

The productions, applicable to words of type I belong to (20), (21), (22), (23), (24) or (25) and then yield words of type I, type I, type J, type I, type L or type J respectively, where type L is defined as follows.

A word is of type L if it is of the form $N_L x \vec{M}_3 y R_1$, with $x \in \Sigma^*$, $y \in \Sigma'^+$, $xh(y) \in K$ and $|xh(y)| \geqq 8$.

The productions, applicable to words of type J belong to (7), (8), (18), (19), (20) or (23) and then yield either a word of type J or type D.

The productions, applicable to words of type L come from (26) or (27) and then yield words of type M or J respectively, where type M is defined as follows. A word is of type M if it is of the form $N_L x \overleftarrow{M}_3 y R_1$ with $x \in \Sigma^*$, $y \in \Sigma'^+$, $xh(y) \in K$ and $|xh(y)| \geqq 8$.

The only productions, applicable to a word of type M come from (28) through (32) and they lead in a deterministic way to $xh(y)$ if the word, they were applied to, was $N_L x \overleftarrow{M}_3 y R_1$.

The above reasoning shows that $L(G) \subseteq K$.

That $K \subseteq L(G)$ can be seen as follows.

If $x \in K$ and $|x| < 8$, then $S \underset{G}{\Rightarrow} x$ and hence $x \in L(G)$.

If $x \in K$ and $|x| \geqq 8$, say $x = a_1 \dots a_k$, $a_i \in \Sigma$ for $1 \leqq i \leqq k$ and $k \geqq 8$, then $S \underset{G}{\Rightarrow} L\vec{M}_0 S' R \overset{*}{\underset{G}{\Rightarrow}} L\vec{M}_0 y R$, with $y \in K'$ and $h(y) = x$ and $L\vec{M}_0 y R \overset{*}{\underset{G}{\Rightarrow}} Ly\vec{M}_0 R \overset{*}{\underset{G}{\Rightarrow}} Ly\overleftarrow{M}_0 R \overset{*}{\underset{G}{\Rightarrow}} x$. Thus $x \in L(G)$. We conclude $K \subseteq L(G)$.

We end the proof by showing that G is nonblocking. To this aim we have to show that for each $w \in \underline{\text{sent}}\, G$, there exists a $\bar{w} \in L(G)$ such that $w \overset{*}{\underset{G}{\Rightarrow}} \bar{w}$. From the proof that $L(G) \subseteq K$ it should be clear that it suffices to prove that each word of $\underline{\text{sent}}\,(G)$ which is of type A through M can lead to a terminal word. For words of types C and M this was already proved in the above. Inspecting the productions of G, we make the following observations. Let $w \in \underline{\text{sent}}\, G$.

If w is of type A, then $w \overset{*}{\underset{G}{\Rightarrow}} w'$ for a w' of type B.

If w is of type B, then $w \overset{*}{\underset{G}{\Rightarrow}} w'$ for a w' of type D.

If w is of type E, then $w \overset{*}{\underset{G}{\Rightarrow}} w'$ for a w' of type F.

If w is of type F, then $w \overset{*}{\underset{G}{\Rightarrow}} w'$ for a w' of type D.

If w is of type $\hat{G}$, then $w \overset{*}{\underset{G}{\Rightarrow}} w'$ for a w' of type H.

If w is of type H, then $w \overset{*}{\underset{G}{\Rightarrow}} w'$ for a w' of type I.

If w is of type I, then $w \overset{*}{\underset{G}{\Rightarrow}} w'$ for a w' of type J or L.

If w is of type J, then $w \overset{*}{\underset{G}{\Rightarrow}} w'$ for a w' of type D.

If w is of type L, then $w \overset{*}{\underset{G}{\Rightarrow}} w'$ for a w' of type J or M.

Hence for each $w \in \underline{\text{sent}}\, G$ of type A, B, D through M, there exists a $w' \in \underline{\text{sent}}\, G$ such that $w \overset{*}{\Rightarrow} w'$ and w' is either of type D or of type M.

Since each word of $\underline{\text{sent}}\, G$ of type M can derive a word of K, it remains to show that each word of $\underline{\text{sent}}\, G$ of type D can derive a terminal word.

This is seen as follows. Let $w \in \underline{\text{sent}}\, G$ and w is of type D. Then $w \overset{*}{\underset{G}{\Rightarrow}} N_L N^i L_1 \overrightarrow{M}_1 S' R_1$ for some $i > 0$. Since K is infinite, K' is also infinite. Hence there is a word $x = a_1 \dots a_k$, with $a_j \in \Sigma'$, $1 \leqq j \leqq k$, such that $x \in K'$ and $k \geqq i+4$. Then

$$N_L N^i L_1 \overrightarrow{M}_1 S' R_1 \overset{*}{\underset{G}{\Rightarrow}} N_L N^i L_1 \overrightarrow{M}_1 x R_1 \overset{*}{\underset{G}{\Rightarrow}} N_L N^i L_1 x \overrightarrow{M}_1 R_1 \overset{*}{\underset{G}{\Rightarrow}} N_L N^i L_1 x \overleftarrow{M}_1 R_1 \overset{*}{\underset{G}{\Rightarrow}}$$

$$\overset{*}{\underset{G}{\Rightarrow}} N_L N^i L_1 \overleftarrow{M}_1 x R_1 \overset{*}{\underset{G}{\Rightarrow}} N_L N^{i+1} M_2 x R_1 \overset{*}{\underset{G}{\Rightarrow}} N_L M_2 \overrightarrow{N}^{i+1} a_1 a_2 \dots a_{i+1} a_{i+2} a_{i+3} \dots a_k R_1$$

$$\overset{*}{\underset{G}{\Rightarrow}} N_L M_2 h(a_1 \dots a_{i+1}) a_{i+2} a_{i+3} \dots a_k R_1 \overset{*}{\underset{G}{\Rightarrow}} N_L \overrightarrow{M}_2 h(a_1 \dots a_{i+1}) a_{i+2} a_{i+3} \dots a_k R_1$$

$$\overset{*}{\underset{G}{\Rightarrow}} N_L h(a_1 \dots a_{i+1}) \overrightarrow{M}_2 a_{i+2} a_{i+3} \dots a_k R_1 \overset{*}{\underset{G}{\Rightarrow}} N_L h(a_1 \dots a_{i+2}) \overrightarrow{M}_3 a_{i+3} \dots a_k R_1$$

$$\overset{*}{\underset{G}{\Rightarrow}} N_L h(a_1 \dots a_{i+2}) \overleftarrow{M}_3 a_{i+3} \dots a_k R_1 \overset{*}{\underset{G}{\Rightarrow}} N_L \overleftarrow{M}_3 h(a_1 \dots a_{i+2}) a_{i+3} \dots a_k R_1$$

$$\overset{*}{\underset{G}{\Rightarrow}} X_1 X_2 h(a_1 \dots a_{i+2}) a_{i+3} \dots a_k R_1 \overset{*}{\underset{G}{\Rightarrow}} h(a_1 \dots a_{i+2}) X_1 X_2 a_{i+3} \dots a_k R_1 \overset{*}{\underset{G}{\Rightarrow}} h(a_1 \dots a_k).$$

Since $a_1 \dots a_k \in K'$, $h(a_1 \dots a_k) \in K$ and hence w derives a word of K.

Thus G is a nonblocking context-sensitive grammar such that $L(G) = K$. Hence the lemma holds. □

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Abstract

A rewriting system G is called *nonblocking* if every sentential form of it can be rewritten into a word of the language of G; otherwise G is called *blocking*. The blocking facility is often used in generating languages by rewriting systems (for example in context-sensitive grammars and EOL systems). This paper initiates the formal investigation of the role that the nonblocking restriction has on the language generating power of various classes of rewriting systems. We investigate grammars of the Chomsky hierarchy as well as context independent L systems with and without tables.

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