Run length control in simulations and performance evaluation and elementary Gaussian processes

By M. Arató[1]

1. Introduction

This paper discusses some statistical problems which arise in analyzing the results of experiments involving the measurement evaluation and comparison of the performance of computing systems, and simulation of such processes, as well. These sequences are generally correlated and in most cases contain a portion which is nonstationary. It is widely accepted that a computer system is operating under a stochastic load and generates stochastic response sequences which are assumed stationary. Such sequences include system response times, utilizations, throughputs (measured e.g. in transactions/sec.), device waiting times, etc. The properties of these output sequences are unknown and the system is being measured in order to estimate characteristics of the specific sequences. As an example the experimenter might be interested in the mean, covariance function of the response times (or even in the response time distribution) and in the utilizations of the major system components (CPU, memory, disks, etc.). Furthermore, the experimenter is often interested in estimating the above quantities as a function of some input parameter such as the number of terminals or transaction rate and in comparing these estimated functions for alternative system configurations. The output sequences are correlated (often strongly) and hence the usual statistical procedures which assume independent observations do not apply.

Let us consider a database system (see e.g. [8], [9]), where transaction response time and transaction rate are particularly important. These have been chosen as the major criteria for evaluating an alternative system. There were made modifications to the operating system so that certain supervisory functions which account for a substantial amount of processor utilization are executed on a separate processor.

A typical time series of transaction response times and its sample correlation function is given in Figure 1.

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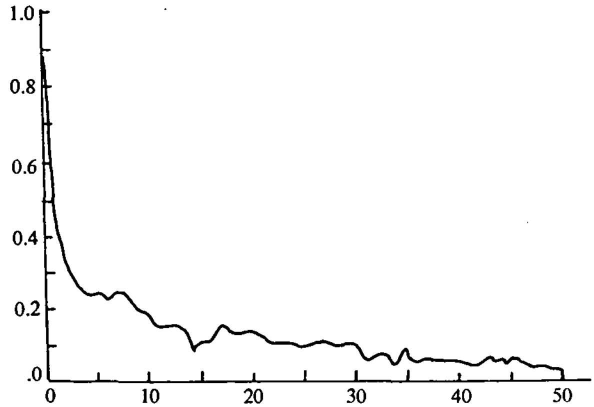

Fig. 1
Sample covariance function of transaction response time

The problem of getting confidence intervals for the mean of a stationary output sequence from a discrete event simulation has an upgrowing literature and program packages (see e.g. [9], [10], [12] and [14]). This problem is connected with a run length control procedure which is designed to terminate the simulation when a confidence interval of a prespecified relative width has been generated or to continue the run to a maximum length.

This paper is concerned with the above mentioned problems for the following practical point of view. Instead of using the spectral analysis techniques, which assume indirectly the asymptotic normality, we are using the stochastic difference and differential equation method, which enables us to calculate the confidence limits in advance, to get exact results in the Gaussian case and, at the same time, good approximations for non-Gaussian sequences.

The results are in good agreement with those of the simulation (see [9], [10]), though the calculations can be carried out on a small calculator, using the tables of the known exact distribution of the maximum likelihood estimator of the damping parameter of an autoregressive (AR) process.

There exist many approaches to the problem of generating confidence intervals for the mean of dependent sequences of random variables and for determining the length of a steady-state simulation. In our method we get the same results by simple calculations based on the concept of sufficient statistics and on the approximation of discrete time process by continuous time process. It is remarkable that explicit results can be gotten and carried out only in the continuous time case.

The main novelty in our method is not only its simplicity, but in the direct estimation of the correlation and giving sufficient statistics. Indeed, instead of the tedious calculations of spectral densities we are using only the first covariances and the boundary random variables which keep the storage requirements of the method extremely low.

Using two estimates

$$\overline{X}_N = \frac{1}{N} \sum_{1}^{N} X_i, \quad \overline{X}_0 = \frac{X_1 + X_N}{2} \tag{1}$$

for the unknown mean $\mu=EX_i$ in the correlated case it is not known which of them is better. Let, for simplicity, X_i be the following time series $X_i=y_i+\mu$, where

$$y_i = \varrho y_{i-1}+\varepsilon_i, \quad (E\varepsilon_i = 0,\ \sigma_\varepsilon^2 = (1-\varrho^2)\sigma_y^2). \tag{2}$$

Then $\bar{X}_N$ is not uniformly, in $0<\varrho<1$, a better estimate than $\bar{X}_0$, in the sense that $\operatorname{Var}\bar{X}_N \geqq \operatorname{Var}\bar{X}_0$, if $\varrho\sim 1$ (see [3], [4]) and compare with (10) below.

Finally, let us point out that in constructing confidence bounds by the spectral method and by the normal approximation, one can find a gap in the earlier proofs, because the authors do not care about the question of uniform (in $0<f_x(0)<\infty$, where $f_x(\lambda)$ is the spectral density of process x) normal approximation when the number of observations $N\to\infty$. Nevertheless, it can easily be seen that uniform approximation does not hold even in the above mentioned special case (2), if $0\leqq\varrho<1$ (see [2], [4]).

2. Preliminary results

The sample covariance functions of waiting time and response time experiments show an exponentially decaying and never an oscillating character, which allows us not to be interested in checking hidden periodicities. In this case, all the roots of the characteristic equation of a higher order AR process are real and negative (in the continuous time case), or less than, in moduli, 1 (in the discrete time case).

This makes possible to assume that the process or one of his derivatives has a simple structure. Our method can be used for higher order autoregressive schemes too, after simple transformations and assuming that the roots of the characteristic polynomial are real.

On the basis of the sample covariance function we may assume that the sequence of observations $X(1), X(2), \dots$ forms a realization of a one dimensional stationary, Markovian and Gaussian process $\xi(n)$ (called elementary Gaussian), with unknown parameters $\mu=E\xi(n)$, $\sigma_\xi^2=D^2\xi(n)=\operatorname{Var}\xi(n)$ and

$$\operatorname{corr}\big(\xi(n), \xi(n-1)\big) = \varrho, \quad \text{i.e.,}$$

$$\big(\xi(n)-\mu\big) = \varrho\big(\xi(n-1)-\mu\big)+\varepsilon(n), \tag{3}$$

where $\varepsilon(n)$ is a Gaussian white noise with $E\varepsilon(n)=0$, $\sigma_\varepsilon^2=(1-\varrho^2)\sigma_\xi^2$.

We are interested for instance in the construction of confidence limits for the parameter μ, or if we denote the process of the base system by $\xi_1(n)$ and the alternative system, after certain functional redistribution by $\xi_2(n)$ then the main question is that whether the difference of sample means

$$\bar{X}_{N,1}-\bar{X}_{N,2}$$

differs significantly from 0 or not. N is the sample size and

$$\bar{X}_{N,i} = \frac{1}{N}\sum_{n=1}^{N} X_i(n), \quad i = 1, 2. \tag{4}$$

Let us recall the following results (see e.g. [4] or [13]). The spectral density function, $f_\xi(\lambda)$, of the process $\xi(n)$ has the form

$$f_\xi(\lambda) = \frac{1}{2\pi}\frac{\sigma_\varepsilon^2}{|1-\varrho e^{-i\lambda}|^2} = \frac{(1-\varrho^2)\sigma_\xi^2}{2\pi}\frac{1}{(1-\varrho\cos\lambda)^2+\varrho^2\sin^2\lambda}, \tag{5}$$

$$f_\xi(0) = \frac{\sigma_\xi^2}{2\pi}\cdot\frac{1+\varrho}{1-\varrho}, \quad 0 \leqq \varrho < 1.$$

If ϱ and σ_ξ^2 are known the maximum likelihood estimator of μ is the following $\left(\text{where } \frac{X_1+X_N}{2}, \sum_1^N X_i \text{ form a system of sufficient statistics}\right)$,

$$\hat{\mu} = \frac{x_1+x_N+(1-\varrho)\sum_2^{N-1} x_i}{2+(1-\varrho)(N-2)}, \tag{6}$$

which is normally distributed with parameters

$$\left(\mu, \sigma_\xi^2\frac{1+\varrho}{2+(1-\varrho)(N-2)}\right). \tag{7}$$

Assuming that $\xi(n)$ is the discrete variant of the continuous process $\xi(t)$ with the differential

$$d\xi(t) = -\lambda\xi(t)\,dt+\sigma_w\cdot dw(t), \quad \varrho = e^{-\lambda\Delta t}, \tag{8}$$

where $w(t)$ is the standard Wiener process, then it is known that σ_w can be estimated exactly and $2\lambda\sigma_\xi^2=\sigma_w^2$. The damping (or decaying) parameter λ (and so ϱ, too) can be estimated poorly and this is the reason why μ has fairly wide confidence intervals. The maximum likelihood estimator of λ is approximately normally distributed if $\lambda T \cong 1000$. Tables of the distribution of the maximum likelihood estimator of the parameter λ can be found in [4], or [5], [6]. In the continuous time case the sufficient statistics of the unknown parameter μ are $\xi(0)+\xi(T)$, $\int_0^T \xi(t)\,dt$ and the maximum likelihood estimator has the form

$$\hat{\mu} = \frac{\xi(0)+\xi(T)+\lambda\int_0^T \xi(t)\,dt}{2+\lambda T}, \tag{9}$$

with variance $2\sigma_\xi^2/(2+\lambda T)$. Note that for $T=1$, $\sigma_w^2=1$ we have

$$D^2\left(\frac{\xi(0)+\xi(1)}{2}\right) = \frac{1+e^{-\lambda}}{4\lambda} < D^2\left(\int_0^1 \xi(t)\,dt\right) = \frac{\lambda+e^{-\lambda}-1}{\lambda^3}, \quad \text{if } \lambda < 2 \tag{10}$$

i.e., depending on λT the mean of two observations can be a better estimate for μ than $\frac{1}{T}\int_0^T \xi(t)\,dt$, and of course better than $\frac{1}{N+1}\sum_{i=0}^N \xi\left(\frac{Ti}{N}\right)$.

The sufficient statistics for λ are

$$s_1^2 = \frac{1}{2}[\xi^2(0)+\xi^2(T)], \quad s_2^2 = \frac{1}{T}\int_0^T \xi^2(t)\,dt, \tag{11}$$

and the maximum likelihood estimator has the form ($\sigma_w^2=1$)

$$\hat{\lambda} = \frac{-[s_1^2-T/2]+\sqrt{[s_1^2-T/2]^2+Ts_2^2}}{2Ts_2^2}. \tag{12}$$

3. Confidence interval construction

Using advantage of the table given in [5] (or [6]) and the approximate variance of $\hat{\mu}$ getting from (7)

$$\frac{\sigma_\xi^2}{N}\frac{1+\varrho}{1-\varrho}, \tag{13}$$

the following approximate confidence intervals can be used for μ/σ_ξ, having the upper $\hat{\varrho}_{.95}$ and lower $\hat{\varrho}_{.05}$ confidence bounds for ϱ at the levels 0.95 and 0.05

$$-1.645\sqrt{\frac{1+\hat{\varrho}}{N(1-\hat{\varrho})}} < \frac{\mu}{\sigma_\xi}-\frac{\hat{\mu}}{\sigma_\xi} < 1.645\sqrt{\frac{1+\hat{\varrho}}{N(1-\hat{\varrho})}} = \tilde{\sigma}_{\hat{\varrho}}(\cdot 9), \tag{14}$$

and $\tilde{\sigma}_{\hat{\varrho}}(0.9)$ we call the half confidence interval width at the level $p=0.9$.

Table 1 contains the lower and upper estimates of ϱ for different sample size and the half confidence interval width at level $p=0.9$ and for all the values $\varrho, \hat{\varrho}_{.95}, \hat{\varrho}_{.05}$.

From Table 1, one can get estimation for the run length control too, in the sense that the required half-width is attained or not. At given ϱ and ε (half-width) with $\varrho_{.05}$ one can get the maximum value $\underline{N}(\varrho)$ for which

$$1.645\sqrt{\frac{1+\hat{\varrho}_{.05}}{N(1-\hat{\varrho}_{.05})}} < \varepsilon, \tag{15}$$

and the minimal value $\overline{N}(\varrho)$

$$1.645\sqrt{\frac{1+\hat{\varrho}_{.95}}{N(1-\hat{\varrho}_{.95})}} < \varepsilon, \tag{16}$$

e.g. for $\varrho=.99=1-\frac{1}{100}$ and $\varepsilon=0.33$ (when $N=5000$) one can get

$$\underline{N}\left(1-\frac{1}{100}\right) = 4320, \quad \overline{N}\left(1-\frac{1}{100}\right) = 7680.$$

Note that in the case when ϱ, σ, m are all unknown, it does not exist such a statistic with known distribution as Student's t in the independent observation case. With this respect we recall the following results (see [2], [3], [4]).

Let us assume for simlicity that $T=1$ and $\sigma_w=1$. Let us take a positive functional $\varkappa(\xi)$ for the lower confidence limit of λ, and $\bar{\mu}(\xi)$, $\underline{\mu}(\xi)$ real-valued functionals as upper and lower confidence limits for μ. We assume that all these func-

Table 1

N	100	500	1000	5000	10000	50000
$\varrho=\mathbf{0.98}$						
λ	2.020	10.101	20.203	101.014	202.03	1010.14
$\hat{\varrho}_{.95}$	0.9996	0.995	0.991	0.985	0.984	0.981
$\hat{\varrho}_{.05}$	0.956	0.969	0.971	0.976	0.977	0.979
$\hat{\sigma}_{.9}(\varrho)$	1.673	0.732	0.518	0.231	0.164	0.979
$\hat{\sigma}_{.9}(\hat{\varrho}_{.95})$	11.630	1.469	0.774	0.268	0.183	0.075
$\hat{\sigma}_{.9}(\hat{\varrho}_{.05})$	1.097	0.586	0.429	0.211	0.153	0.071
$\varrho=\mathbf{0.99}$						
λ	1.010	5.025	10.050	50.252	100.50	502.52
$\hat{\varrho}_{.95}$	0.9999*	0.9993	0.9976	9.9934	0.9924	0.9911
$\hat{\varrho}_{.05}$	0.9750	0.9816	0.9841	0.9869	0.9879	0.9891
$\hat{\sigma}_{.9}(\varrho)$	2.321	1.038	0.734	0.328	0.232	0.104
$\hat{\sigma}_{.9}(\hat{\varrho}_{.95})$	23.263*	3.032	1.501	0.404	0.266	0.110
$\hat{\sigma}_{.9}(\hat{\varrho}_{.05})$	1.426	0.763	0.581	0.287	0.211	0.099
$\varrho=\mathbf{0.995}$						
λ	0.5012	2.506	5.013	25,063	50.125	250.63
$\hat{\varrho}_{.95}$	0.9999*	0.9999	0.9996	0.9973	0.9967	0.9959
$\hat{\varrho}_{.05}$	0.9852	0.9893	0.9908	0.9928	0.9934	0.9942
$\hat{\sigma}_{.9}(\varrho)$	3.286	1.469	1.039	0.465	0.329	0.147
$\hat{\sigma}_{.9}(\hat{\varrho}_{.95})$	23.263*	23.263	3.678	0.633	0.405	0.162
$\hat{\sigma}_{.9}(\hat{\varrho}_{.05})$	1.905	1.003	0.765	0.387	0.286	0.136
$\varrho=\mathbf{0.998}$						
λ	0.202	1.001	2.002	10.010	20.020	100.10
$\hat{\varrho}_{.95}$	0.9999*	0.9999*	0.9999	0.9995	0.9991	0.9985
$\hat{\varrho}_{.05}$	0.9925	0.9950	0.9955	0.9968	0.9971	0.9975
$\hat{\sigma}_{.9}(\varrho)$	5.199	2.325	1.644	0.735	0.520	0.233
$\hat{\sigma}_{.9}(\hat{\varrho}_{.95})$	23.263*	23.263*	23.263	1.471	0.775	0.269
$\hat{\sigma}_{.9}(\hat{\varrho}_{.05})$	2.681	1.469	1.095	0.581	0.432	0.208
$\varrho=\mathbf{0.999}$						
λ	0.100	0.500	1.001	5.003	10.005	50.03
$\hat{\varrho}_{.95}$	0.99999*	0.99999*	0.99999*	0.99993	0.99976	0.99933
$\hat{\varrho}_{.05}$	0.99700	0.99710	0.99748	0.99815	0.99840	0.99869
$\hat{\sigma}_{.9}(\varrho)$	7.35	3.289	2.326	1.040	0.735	0.329
$\hat{\sigma}_{.9}(\hat{\varrho}_{.95})$	73.566*	73.566*	73.566*	3.932	1.502	0.402
$\hat{\sigma}_{.9}(\hat{\varrho}_{.05})$	4.244	1.931	1.410	0.765	0.581	0.287

The half confidence interval width $\hat{\sigma}_p(\varrho)=1.645\sqrt{(1+\varrho)/N(1-\varrho)}$ at level p for $\mu/\sigma_\xi\cdot$ $\hat{\varrho}_\beta$ means the β level confidence bound of ϱ, $\varrho=e^{-\lambda/N}$, $\lambda=-N\log\varrho$, N is the sample size.

* In the cases marked by * the upper confidence bound for ϱ is equal to 1 and the confidence interval width is ∞ (see section 4).

tionals are continuous on R_ξ in the $C[0, 1]$ metric, but $\bar{\mu}$ and $\underline{\mu}$ may assume values $+\infty$ and $-\infty$. The continuity of functionals assuming infinite values is to be understood as continuity induced by the topology of the real line, closed by points $-\infty$ and ∞. First we have the following assertion, which says that no nonzero lower limit can be constructed for the parameter λ with any degree of confidence.

Theorem 1. *Let* $\beta>0$, *and let* $\varkappa(\xi)$ *be a positive functional defined in the space* R_ξ *and continuous in the* $C[0, 1]$ *metric, with the property that* $\varkappa(\xi)\to\infty$ *if* $\sup|\xi(t)|\to\infty$. *Let it satisfy for any* μ *and* λ *the condition* $P\{\lambda>\varkappa(\xi)\}>\beta$. *Then*

$$P\{\varkappa(\xi)=0\}\geqq g(\lambda,\beta) \tag{17}$$

where the positive function $g(\cdot)$ *does not depend on the choice of functional and* $g(\lambda,\beta)\to 1$ *as* $\lambda\to 0$.

For parameter μ the following statement says that if μ, λ are unknown it is impossible to construct finite confidence intervals using continuous functions. We assume that $\bar{\mu}$ and $\underline{\mu}$ has the property that for a real value c

$$\bar{\mu}(\xi+c)=\bar{\mu}(\xi)+c, \quad \underline{\mu}(\xi+c)=\underline{\mu}(\xi)+c. \tag{18}$$

Theorem 2. *Let* $\beta>1/2$, *and let* $\underline{\mu}(\xi), \bar{\mu}(\xi)$ *be real valued functionals (which may assume values* $-\infty$ *or* $+\infty$*) on the space* R_ξ, *which are continuous in the* $C[0, 1]$ *metric and which satisfy the conditions*

$$\begin{aligned} &P\{\mu\geqq\underline{\mu}(\xi)\}\geqq\beta,\\ &P\{\mu<\bar{\mu}(\xi)\}\geqq\beta, \end{aligned} \tag{19}$$

for any μ *and* $\lambda(-\infty<\mu<\infty, \lambda>0)$. *Then*

$$\begin{aligned} &P\{\bar{\mu}(\xi)=\infty\}\geqq f(\lambda,\beta),\\ &P\{\underline{\mu}(\xi)=-\infty\}\geqq f(\lambda,\beta), \end{aligned} \tag{20}$$

where $f(\lambda,\beta)$ *does not depend on the choice of these functionals, and* $f(\lambda,\beta)\to 1/2$ *as* $\lambda\to 0$.

Simulation results were given in [6] to illustrate the situation and to have a picture on the function $g(\lambda,\beta)$, where the following estimators $(T=1, \sigma_w^2=1)$ were taken:

$$\tilde{m}_1=\frac{1}{N}\sum_1^N\xi_i, \quad \tilde{\lambda}_1=\frac{1}{\frac{2}{N}\sum_1^N(\xi_i-\tilde{m}_1)^2},$$

$\tilde{m}_2, \tilde{\lambda}_2$ the maximum likelihood estimators,

$$\tilde{m}_3=\frac{\xi(0)+\xi(1)}{2}, \quad \tilde{\lambda}_3=\frac{2}{(\xi(1)-\xi(0))^2},$$

where $\xi_1=\xi(i/N)$, $(i=1, 2, \ldots, N)$, $\xi_0=\xi(0)$. N was taken between 60 and 100 and n (the number of samples) was 1000. We have the following approximations:

$$\begin{aligned}
g(\lambda, 0.05) &\approx 1 \quad \text{if} \quad \lambda<0.5, \text{ (i.e., } P(\varkappa(\xi)=0)=1 \text{ if } \hat{\lambda}\leqq 0.5 \text{ on level } \beta=0.05),\\
g(\lambda, 0.05) &\approx 1 \quad \text{if} \quad \lambda<4,\\
g(\lambda, 0.9) &\approx 1 \quad \text{if} \quad \lambda<9,\\
g(\lambda, 0.95) &\approx 1 \quad \text{if} \quad \lambda<12.
\end{aligned}$$

It seems that

$$g(\lambda, \beta) \approx e^{-c_\beta \lambda}, \quad \text{when} \quad \lambda \to 0,$$

but this statement is not proved.

Theorem 2 can be reworded as follows: *When the parameters μ and λ of a stationary Gaussian Markov process are unknown, it is impossible to construct finite confidence intervals for μ using continuous functionals.*

From the proof provided in [2] it can be seen that for any $\varepsilon>0$ there exists a $\Lambda(\varepsilon)$ such that for small values of λ

$$\sup_{\lambda<\lambda_0, \mu} P_{\lambda,\mu}\{\bar{\mu}(\xi) > \mu\} \leqq 1/2+\Lambda \cdot \lambda_0^{(1/2)-\varepsilon}.$$

4. Run length control and sequential estimation

Running a simulation less than its length would not provide the information needed, while running it longer would be a waste of time, so it has great practical meaning for the experimenter to have some preliminary estimation about the accuracy requirements. We shall assume further, that this accuracy requirement is specified by the half-width of the confidence interval of the mean value, μ, devided by the standard deviation, σ_ξ, of the process $\xi(t)$. In this section we will describe the incorporation of the method of sections 2 and 3 into a sequential estimation procedure. We shall show that one possible approach is that, when using the approximation with continuous time we estimate the decay parameter λ (and so ϱ) by given accuracy. This procedure uses the same amount of storage required earlier but uses some new random time moments (the Markov moments) and requires only a small amount of computing per output element.

Let us denote by ε the required relative half-width of the ratio μ/σ_ξ, and by $p=1-\beta$ the given confidence level, and $x_{1-(\beta/2)}$ the $1-(\beta/2)$-quantile of the Gaussian distribution.

For given α, where $1-\alpha$ means the confidence level for ϱ, to make small the difference

$$\frac{x_{1-(\beta/2)}}{\sqrt{N}}\left[\sqrt{\frac{1+\hat{\varrho}_{1-(\alpha/2)}}{1-\hat{\varrho}_{1-(\alpha/2)}}}-\sqrt{\frac{1+\hat{\varrho}_{\alpha/2}}{1-\hat{\varrho}_{\alpha/2}}}\right] < \varepsilon_1, \tag{21}$$

we shall take advantage of sequential estimation of ϱ. For given α and c let us take H in such a way that (x_α denotes the α quantile of normal distribution).

$$H < \frac{c^2}{(x_{1-(\alpha/2)}-x_{\alpha/2})^2}. \tag{22}$$

Further, let us denote by

$$\tau(H) = \inf\left\{t\colon \int_0^t \xi^2(s)\,ds \geqq H(\varepsilon, \alpha)\right\}, \tag{23}$$

the Markov moment and take

$$\lambda(H) = -\frac{1}{H}\int_0^{\tau(H)} \xi(t)\,d\xi(t) = -\frac{\xi^2(\tau(H))-\xi^2(0)-\tau(H)}{2\int_0^{\tau_H} \xi^2(t)\,dt}. \tag{24}$$

Then the following statement is true (see LIPTSER, SHIRYAEV [13], ARATÓ [4]).

Theorem 3. *The sequential estimator $\lambda(H)$ is normally distributed with parameters*

$$E_\lambda \lambda(H) = \lambda, \quad D^2(\lambda(H)) = \frac{1}{H}, \tag{25}$$

and it is efficient, i.e., it has minimal variance.

The calculated H depends on C, α and from the realization getting $\tau(H)$ for given ε_1, β it is possible to check (compare with (21)).

$$\frac{x_\beta}{\sqrt{\tau(H)}}\left[\sqrt{\frac{1+\hat{\varrho}_{1-(\alpha/2)}}{1-\hat{\varrho}_{1-(\alpha/2)}}} - \sqrt{\frac{1+\hat{\varrho}_{\alpha/2}}{1-\hat{\varrho}_{\alpha/2}}}\right] < \varepsilon_1, \tag{26}$$

where $\hat{\varrho}=e^{-\hat{\lambda}\cdot\Delta/N}$, $\hat{\lambda}_{1-(\alpha/2)}=\lambda(H)+x_{1-(\alpha/2)}/\sqrt{H}$. After the fulfilment of (26) one can construct confidence limits for the unknown mean μ.

To get some approximations for $\tau(H)$ one has to turn to the papers of NOVIKOV [15]—[17] (see also LIPTSER—SHIRYAEV [13]).

Theorem 3 remains valid (under some natural conditions on $a(t, \xi)$) if we regard the process

$$d\xi(t) = \lambda a(t, \xi(t))\,dt + dw(t),$$

(see LIPTSER—SHIRYAEV [13] § 17.5).

A natural question arises whether the advantages of sequential estimators are consequences of a rather long mean observation time $E_\lambda(\tau(H))$. For general $a(t, \xi)$ this question is unsolved. The following statement is true (see NOVIKOV [17]).

Theorem 4. *For $\lambda \geqq 0$ as $T \to \infty$,*

$$P_\lambda(\tau(H) \geqq T) = 4\left(\frac{H}{\pi T^2}\right)^{1/2} \exp\left\{-\frac{\lambda^2 H}{2} - \frac{T^2}{8H} + \frac{\lambda T}{2}\right\}(1+o(1)), \tag{27}$$

$$E_\lambda \tau(H) \leqq 2[\lambda H + 2\sqrt{H}] + \sqrt{8(\lambda^2 H^2 + 4\lambda H) + 2H}. \tag{28}$$

Further, if $\lambda^2 H \to \infty$, then

$$E_\lambda \tau(H) = 2\lambda H\left(1 + \frac{3}{4\lambda^2 H} + o\left(\frac{1}{\lambda^2 H}\right)^2\right), \tag{29}$$

and if $\lambda^2 H \to 0$, then

$$E_\lambda \tau(H) = H^{1/2}[2.09 + 0.856\lambda H^{1/2} + o(\lambda^2 H)]. \tag{30}$$

Note that remarkable fact that these results are in good agreement of those simulation results which are published in HEIDELBERGER, WELCH [9], [10] or HEIDELBERGER [8].

Tables based on Theorems 3 and 4, one can construct easily.

Abstract

This paper intends to show that the method proposed by Kolmogorov in constructing confidence limits for diffusion type processes gives a more simple and straightforward tool in run length control of output sequences of stationary series than the spectral method. There exists an upgrowing literature of the spectral method for construction confidence limits (see e. g. the survey paper HEIDELBERGER, WELCH [9]), and even software program packages were constructed on this basis. We show that the Gaussian processes, when the computational requirements and storage remain low, can be used as good approximations with the advantage that instead of simulation one can get exact formulas. The connection between run length control and sequential estimation methods are found and some results of Novikov can be used.

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