On some extensions of russian parallel context free grammars

By JÜRGEN DASSOW

1. In the last years some authors have studied the effect of mechanisms regulating the derivation process to the generative capacity. The matrix grammars, programmed grammars, random context grammars, and periodically time-variant grammars belong to the most investigated mechanisms. A. Salomaa, O. Mayer, and M. V. Lomkovskaja, and others have given the results in the case of context free grammars (see [11], [5]); concerning L systems investigations have been done by S. H. von Solms, G. Rozenberg, and P. J. A. Reusch (see [12], [8], [6]); the author has considered extension of indian parallel context free grammars ([1]).

In this note we will complete these results and study the generative capacity of extensions of russian parallel grammars introduced by M. K. Levitina [3]. Already Levitina regarded the extension by the matrix mechanism and proved that the associated family of languages coincides with the family of programmed context free grammars. We will show that also the extensions by the mechanisms of programmed grammars, random context grammars, and periodically time variant grammars generate the same family of languages.

2. For sake of completeness, we will recall some definitions shortly. For detailed information see [11], [3]. A *russian parallel context free* grammar is a construct $G=(V_T, V_N, P, S)$ where

i) V_T and V_N are disjoint nonempty finite sets, $V=V_N\cup V_T$,

ii) P is a finite set of pairs (p, i) where $i\in\{1, 2\}$ and p is a production $A\to w$ with $A\in V_N$, $w\in V^+$,

iii) $S\in V_N$.

The derivation $x\Rightarrow y$, $x, y\in V^+$, is defined by

i) $x=x_1Ax_2$, $y=x_1wx_2$, $x_1, x_2\in V^*$,

ii) $(A\to w, 1)\in P$

or by

i) $x=x_1Ax_2Ax_3\dots x_{n-1}Ax_n$, $y=x_1wx_2wx_3\dots x_{n-1}wx_n$, $x_1, x_2, \dots, x_n\in(V\setminus\{A\})^*$,

ii) $(A\to w, 2)\in P$.

$\overset{*}{\Rightarrow}$ denotes the reflexive and transitive closure of $\Rightarrow$. The language $L(G)$ generated by G is defined as

$$L(G) = \{w: S \overset{*}{\Rightarrow} w, \; w \in V_T^*\}.$$

Now we will give some mechanisms regulating the derivation process.

Programmed grammars: Each rule has the form $(l: (A \to w, i), F, S)$ where l is the label of the rule, F and S (the failure field and the success field) are sets of labels. If A occurs in x, we rewrite x (as in a russian parallel grammar), and in the next derivation step we have to apply a rule with a label in S. If A does not occur in x, we apply a rule with a label in F.

Random context grammars: Each rule has the form $((A \to w, i), U, T)$ where U and T are subsets of V_N. The production $(A \to w, i)$ is applicable to x if and only if any symbol of U occurs in x and no symbol of T occurs in x.[1]

Periodically time variant grammars: We associate a subset $\varphi(i)$ of P with an integer $i \geqq 1$ such that, for each k, $\varphi(n+k+j) = \varphi(n+k)$ for some n and j. The rule applied in the i-th step of the derivation has to be choosen from the set $\varphi(i)$.

For these three type of grammars, the generated language is defined as above.

Matrix grammars: A matrix $m_i = [r_{i1}, r_{i2}, \ldots, r_{i,i_k}]$ is an ordered sequence of rules $r_{i,j} \in P$. The application of a matrix m_i to a word x is defined as the application of the rules r_{ij} in the given order. The generated language consists of all words over V_T which can be derived from S by applications of matrices.

We use the following notations:

$\mathcal{F}(PRP)$ — family of programmed russian parallel languages,
$\mathcal{F}(RCRP)$ — family of random context russian parallel languages,
$\mathcal{F}(TVRP)$ — family of russian parallel periodically time-variant languages,
$\mathcal{F}(MRP)$ — family of russian parallel matrix languages.

If $i=1$ $(i=2)$ for all rules $(A \to w, i)$ of P, we get the context free grammars (indian parallel context free grammars, introduced by K. Krithivasan and R. Siromoney) and its associated extensions. We will use the letters CF or IP instead of RP to denote the corresponding family of languages. It is known that

$$\mathcal{F}(PCF) = \mathcal{F}(RCCF) = \mathcal{F}(TVCF) = \mathcal{F}(MCF),$$

$$\mathcal{F}(PIP) = \mathcal{F}(RCIP) = \mathcal{F}(TVIP) = \mathcal{F}(MIP),$$

and

$$\mathcal{F}(XIP) \subset F(XCF) \quad \text{for} \quad X \in \{P, RC, TV, M\}.$$

3. We will prove analogous relations for russian parallel versions, too.

Theorem. $\mathcal{F}(PRP) = \mathcal{F}(RCRP) = \mathcal{F}(TVRP) = \mathcal{F}(MRP) = \mathcal{F}(PCF)$.

Proof i) By definition, $\mathcal{F}(XCF) \subseteq \mathcal{F}(XRP)$ for $X \in \{P, RC, TV, M\}$. Therefore we have to prove $\mathcal{F}(XRP) \subseteq \mathcal{F}(XCF)$ only.

ii) $\mathcal{F}(PRP) \subseteq \mathcal{F}(PCF)$.

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[1] This definition is due to Lomkovskaja and differs slightly from van der Walt's definition. The difference has no effect to the generative capacity. By the parallel rewriting (if $i=2$), the above definition is more useful.

Let $L=L(G)$ for the programmed russian parallel grammar $G=(V_T, V_N, P, S)$. We will construct a programmed context free grammar G' which simulates the application of rules $(A\to w, 2)$ by a set of usual context free productions (in the construction we will write only $B\to v$ instead of $(B\to v, 1)$). We put

$$V' = \{A_l:(l:(A\to w, 2), F, S)\in P\},$$
$$P_1 = \{(l:A\to w, F, S):(l:(A\to w, 1), F, S)\in P\},$$
$$P_2 = \{(l:A\to A_l, F, \{l'\}):(l:(A\to w, 2), F, S)\in P\},$$
$$P_3 = (l':A\to A_l, \{l''\}, \{l'\}):(l:(A\to w, 2), F, S)\in P,$$
$$P_4 = \{(l'':A_l\to w, S, \{l''\}):(l:(A\to w, 2), F, S)\in P\},$$

and $G'=(V_T, V_N\cup V', P_1\cup P_2\cup P_3\cup P_4, S)$. Obviously, G' is a programmed context free grammar with $L(G')=L$.

iii) $\mathcal{F}(RCRP)\subseteq\mathcal{F}(RCCF)$.

Let $L\in\mathcal{F}(RCRP)$ and $L=L(G)$ for some random context russian parallel grammar $G=(V_T, V_N, P, S)$. We introduce new alphabets V_1 and V_2 by $V_i=\{A_i\colon A\in V_N\}$, $i=1, 2$, and define the homomorphism h on V by $h(A)=A_2$ for $A\in V_N$ and $h(a)=a$ for $a\in V_T$. Further we put

$$P_1 = \{(A\to w, U, T\cup V_1\cup V_2):((A\to w, 1), U, T)\in P\},$$
$$P_2 = \{(A\to A_1, U, T\cup((V_1\cup V_2)\setminus\{A_1\}):((A\to w, 2), U, T)\in P\},$$
$$P_3 = \{(A_1\to h(w), (U\setminus\{A\})\cup\{A_1\}, (T\cup\{A\}\cup V_1)\setminus\{A_1\}):$$
$$:((A\to w, 2), U, T)\in P\},$$
$$P_4 = \{(A_2\to A, \{A_2\}, V_1):A_2\in V_2\},$$

and

$$G' = (V_T, V_N\cup V_1\cup V_2, P_1\cup P_2\cup P_3\cup P_4, S).$$

If the production $(A\to w, 1)$ is applicable to x in G, then the corresponding production of P_1 is applicable in G', and we derive the same word in both grammars. Now let $(A\to w, 2)$ be applicable to x in G and derive the word y. Then $A\to A_1$ is applicable to x in G', and then we have to apply $A\to A_1$ on all occurrences of A in x. Now we can apply only $A_1\to h(w)$, and we have to do this substitution at all occurrences of A_1. Then we have to use the applicable rules of P_4 and we get also the word y. Therefore $L(G)\subseteq L(G')$. The other inclusion can be proved by analogous arguments. Thus we have constructed a random context (context free) grammar G' with $L=L(G')$.

iv) $\mathcal{F}(TVRP)\subseteq\mathcal{F}(TVCF)$.

Let $L\in\mathcal{F}(TVRP)$ and $L=L(G)$ for some periodically time-variant russian parallel grammar $G=(V_T, V_N, P, S)$. Let $\varphi(i)$ be the subsets of P such that $\varphi(m)=\varphi(m+j)$ for a certain j and all $m\geqq n$. We will construct a programmed russian parallel grammar G' such that $L=L(G')$, which proves $\mathcal{F}(TVRP)\subseteq\subseteq\mathcal{F}(PRP)$. Thus $\mathcal{F}(TVRP)\subseteq\mathcal{F}(TVCF)$ by ii) and the result concerning the context free case.

Let $V_N=\{A_1, \ldots, A_s\}$. We introduce new alphabets $V_i=\{A_{ki}: 1\leq k\leq s\}$ and the homomorphisms h_i on V by

$$h_i(x) = \begin{cases} A_{ki} & \text{if } x = A_k, \\ a & \text{if } x = a\in V_T \end{cases}$$

for $i=1, 2, \ldots, n+j-1$. Further we put

$$\varphi'(i) = \{(A_{ki} \to h_{i+1}(w), r): 1 \leq k \leq s, (A_k \to w, r)\in\varphi(i)\}$$

for $i = 1, 2, \ldots, n+j-2$,

$$\varphi'(n+j-1) = \{(A_{k,n+j-1} \to h_n(w), r): 1 \leq k \leq s, (A_k \to w, r)\in\varphi(n+j-1)\}.$$

We consider the programmed russian parallel grammar $G'=(V_T \bigcup_{i=1}^{n+j-1} V_i, P', S_1)$ where the elements of P are given in figure 1.

Obviously, $L=L(G')$.

v) $\mathscr{F}(MRP)\subseteq\mathscr{F}(MCF)$.

This fact follows from Levitina's result $\mathscr{F}(MRP)=\mathscr{F}(PCF)$. (Using the method of iv) we can prove it.)

Corollary 1. For $X\in\{P, RC, TV, M\}$, $\mathscr{F}(XIP)\subsetneqq\mathscr{F}(XRP)$.

Because some of the properties of $\mathscr{F}(PCF)$ are known, for instance $\mathscr{F}(PCF)$ forms an AFL, we get also information on the extensions of russian parallel languages.

4. A language L is called of index k if there exists a grammar G with $L(G)=L$ such that any word $w\in L$ has a derivation with the property that each sentential form of this derivation contains at most k occurrences of letters of V_N. L is of finite index iff there exists an integer k such that L is of index k.

By $\mathscr{F}(X)_{FIN}$ we denote the family of languages of $\mathscr{F}(X)$ which are of finite index.

By the results of [9], [7], [10], and the fact that the construction in [1] and in this note preserve the finiteness of the index of a language, we get a second corollary.

Corollary 2. $\mathscr{F}(PCF)_{FIN}=\mathscr{F}(RCCF)_{FIN}=\mathscr{F}(TVCF)_{FIN}=\mathscr{F}(MCF)_{FIN}=$
$=\mathscr{F}(PIP)_{FIN}=\mathscr{F}(RCIP)_{FIN}=\mathscr{F}(TVIP)_{FIN}=\mathscr{F}(MIP)_{FIN}=$
$=\mathscr{F}(PRP)_{FIN}=\mathscr{F}(RCRP)_{FIN}=\mathscr{F}(TVRP)_{FIN}=\mathscr{F}(MRP)_{FIN}$.

In [9], properties of this language family are studied. For instance, it forms an AFL again.

5. Finally, we remark that the context free languages and the russian parallel context free languages are incomparable with the extensions of indian parallel languages. This follows by the following facts:

— $\{a^nb^nc^n: n\geq 1\}$ is in $\mathscr{F}(MIP)$, it is not a russian parallel context free language ([3]),

— the extensions of indian parallel context free languages coincide with the

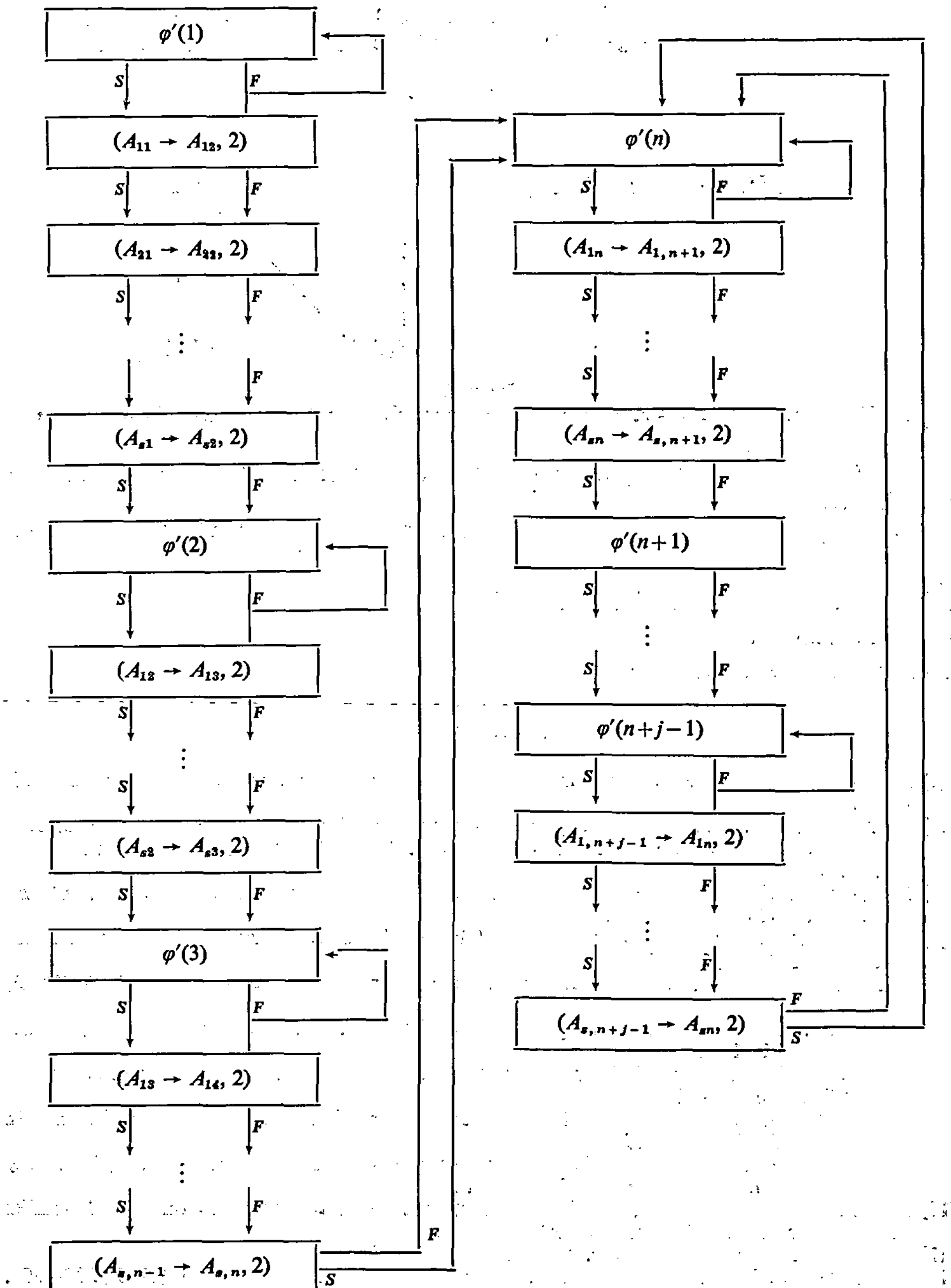

Figure 1

An arrow labelled by S leads to the success field; an arrow labelled by F to the failure field.

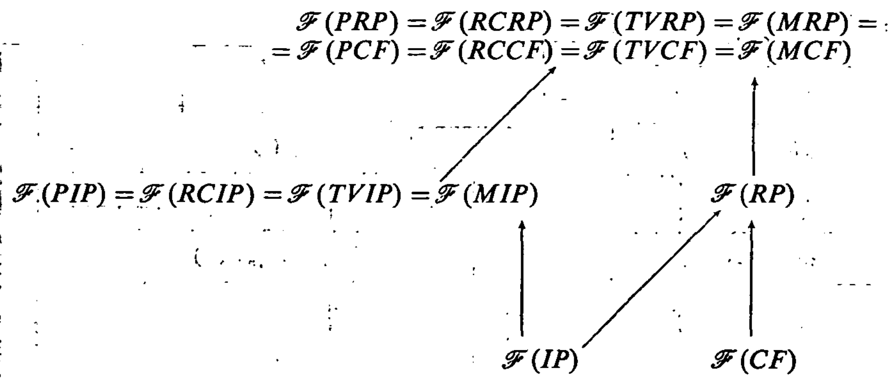

Figure 2

$X \to Y$ denotes $X \subseteq Y$, $X \neq Y$. Language families which are not connected are incomparable.

EDTOL languages ([1]), and there are context free languages which are not *EDTOL* languages.

Thus figure 2 gives the complete relation between the regarded language families.

TECHNOLOGICAL UNIVERSITY OTTO VON GUERICKE
DEPARTMENT OF MATHEMATICS AND PHYSICS
301 MAGDEBURG
BOLESLAW-BIERUT-PL. 5
PSF 124
GERMAN DEMOCRATIC REPUBLIC

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