A solution of the early bird problem in an n-dimensional cellular space

By T. LEGENDI and E. KATONA

The early bird problem has been defined in [1] and has been solved in [1], [2], [3], [5]. In this paper the problem is generalized for the n-dimensional cellular space, and a real time solution is given using $0(mn)$ steps in an n-dimensional cellular field of edge length m. The solution will be shown in detail in the two-dimensional case. The basic idea of the solution is to reduce the n-dimensional early bird problem to the $(n-1)$-dimensional one, using special signals and applying the one-dimensional early bird algorithm from [3].

1. Introduction

Let us consider a cellular space in which any cell in quiescent state may be excited from the outside world. The excitations result in special "bird" states instead of the quiescent states. The task is to give a transition function ensuring after a certain time the first excitation(s) may be distinguished from the later ones. This is the early bird problem defined originally by Rosenstiehl et al. [1] for an elementary cyclic graph where to each of the m vertices an automaton (cell) is assigned. In [1] a $2m$-step solution is given on condition that at each time-step maximum one excitation occurs.

Vollmar [2] defined and solved the problem for a one-dimensional cellular space allowing more than one excitation at each time-step. The solution is based on the "age of waves" concept and requires a high number of cell-states.

In [3] a simplified solution has been given for the one-dimensional case using only 5 cell-states. Kleine Büning [4] proved that the early bird problem is unsolvable with 4 states in a one-dimensional cellular space, that is, the solution in [3] is optimal considering the number of states.

In [5] the early bird problem is solved for a two-dimensional cellular space in nonlinear time (for an $m \cdot m$ space $0(m^2)$ steps are needed.

2. Exact definition of the n-dimensional early bird problem

Let (I^n, S, N, f) be an n-dimensional cellular space, where

I is the set of integers and to each point of I^n a *cell* is assigned;

S is the finite set of *cell-states* containing 3 special states, the passive state $\#$, the quiescent state q and the "bird" state β;

N is the *neighbourhood index*, in this paper the von Neumann neighbourhood index is assumed: $N=((0,0),(0,1),(0,-1),(1,0),(-1,0))$;

f: $S^5 \to S$ is the *local transition function* satisfying $f(q, \dots, q)=q$ and $f(\#, a, b, c, d)=\#$ for any $a, b, c, d \in S$.

In the cellular space an n-dimensional cube K of edge length m is assigned: $K=\{(i_1, \dots, i_n) | 0 \leqq i_j \leqq m-1, j=1, \dots, n\}$. In the initial configuration the cells in K are in state q (active space), all other cells are in state $\#$ (passive space).*

The cellular space works, as usual, in discrete time-steps $t=0, 1, 2, \dots$ using the local transition function f, but between two steps (that is to say, at time $t+1/2$) any quiescent cell (in state q) may change spontaneously into the bird state β (excitation). The problem is to define a transition function f ensuring that after a certain time the bird(s) arisen at first is (are) in a distinguished state, and all other cells in other states.

To simplify the solution, the excitation of the border cells (which have a neighbour in state $\#$) will be prohibited.

For an easier explanation, first the solution will be presented in the two-dimensional case (point 3, 4). The generalization for n-dimensions will be discussed in point 5.

3. The sketch of the solution in the two-dimensional case

The basic idea of the algorithm is that any bird sends out a special signal in horizontal direction, which arrives at the leftmost column z steps after its origin where z is independent of the place of the bird (Fig. 1). That is, any excitation in the two-dimensional field at time t, may generate a "secondary excitation" in the leftmost column at time $t+z$. In this way the two-dimensional early bird problem can be "projected" into a one-dimensional one.

A bird will be called a *local early bird* if there is no earlier bird in its row, and it will be called a *global early bird* if there is no earlier bird in the cellular space. Using a one-dimensional early bird algorithm in each row, the local early birds can be marked. Using a one-dimensional early bird algorithm in the leftmost column, we can decide that which rows contain the global early birds. It is clear that the vertical and the horizontal early bird algorithms together may select the global early birds.

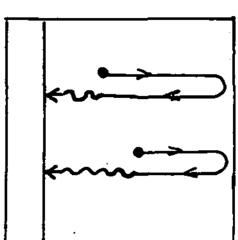

Fig. 1

The main problem in the above solution is to ensure a constant z delay for any signal sent by a bird. The solution of this problem is explained in a time-space diagram (Fig. 2).

* This partition of the cellular space corresponds to the "retina" conception of [7].

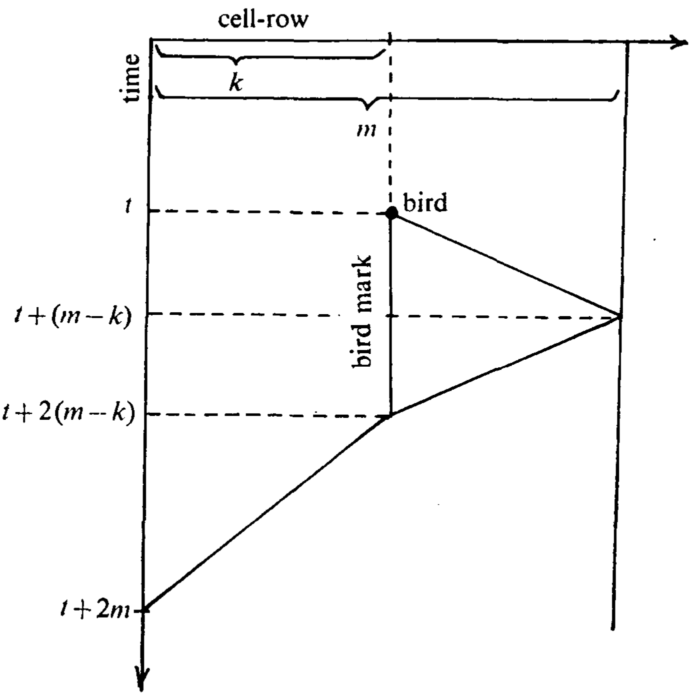

Fig. 2
The time-space diagram of a row of the cellular space

Let us consider a bird arisen at time t in the k-th cell of a cell-row. This bird sends a full-speed signal to the right (the signal steps right in each step) and in the k-th cell there remains a special sign, a so-called "bird mark". The full-speed signal is reflected at the rightmost cell at time $t+(m-k)$ and moves left until it reaches the bird mark (at time $t+2(m-k)$). Here it cancels the bird mark (the bird itself survives), and the full-speed signal alters into a half-speed signal moving to the left. This signal reaches the leftmost cell at time $t+2(m-k)+2k=t+2m$, thus the constant delay of the signal is ensured.

Considering more than one bird, there arises the question whether each full-speed signal will cancel its own bird mark. This question can be answered with "yes" because the following property holds.

Proposition. Let b_1 and b_2 two bird marks in the k_1-th and k_2-th cells of the cell-row, respectively. If $k_1<k_2$ then the full-speed signal s_2 (of b_2) goes before the full-speed signal s_1 (of b_1).

Proof. The definition of the early bird problem contains the restriction that only quiescent cells may alter into the bird state. Considering the 5-state early bird algorithm of [3] it is clear that the cells between the bird mark b_1 and its signal s_1 cannot be in quiescent state, therefore the excitation in this range is prohibited. This fact implies the above proposition. □

It results from the proposition that the first reflected signal has been sent by the rightmost bird, the second reflected signal by the next rightmost bird, etc., thus the cancellation of the bird marks always will be correct.

Summarizing, *the sketch of the two-dimensional algorithm* is as follows:

(i) Any new bird in the active cellular field sends out a signal which arrives at the leftmost column with a constant delay. At the same time, a one-dimensional early bird algorithm is executed in each row which cancels all "local late birds".

(ii) The signals arriving at the leftmost column appear as "secondary birds" and a vertical one-dimensional early bird algorithm is applied among them. If a secondary bird proved to be later then it is killed and a "cancel" signal starts moving to the right in that row which kills all the birds there.

After 6m steps (see the time estimation in point 5) only the global early birds exist in the cellular space.

4. Detailed description of the solution in the two-dimensional case

The state set $S'=S-\{\#\}$ is composed of two components: $S'=S_1\times S_2$. Component S_1 serves for the 5-state early bird algorithm of [3], that is, $S_1=\{Q, B, R, L, N\}$ where the states are named "quiescent", "bird", "right wave", "left wave", "neutral", respectively.

The component S_2 ensures the movement of signals according to Fig. 2. It consists of 5-bit words: $S_2=\{(s_1, \ldots, s_5)|s_i\in\{0, 1\}, i=1, \ldots, 5\}$, where

$s_1=1$ means "bird mark",
$s_2=1$ means "full-speed signal moving to the right",
$s_3=1$ means "full-speed signal moving to the left",
$s_4=1$ means "half-speed signal in phase 1",
$s_5=1$ means "half-speed signal in phase 2".

The quiescent state q and the bird state β (see point 2) are defined as $q=(Q,00000)$, $\beta=(B,11000)$.

The active cellular space is divided into three areas (Fig. 3). The leftmost column is called as area A_1, the inner cells as area A_2 and the rightmost column as area A_3. The transition function f involves three subfunctions f_1, f_2, f_3 according to A_1, A_2, A_3. (The cells in different areas may be distinguished by their left and right neighbours.) In the sequel the areas A_1, A_2, A_3 will be discussed separately.

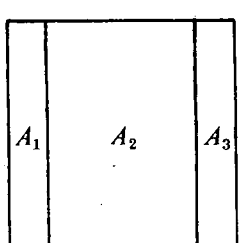

Fig. 3

Area A_2 (inner cells).

The transition function f_2 is composed of two functions. For the component S_1 the 5-state early bird function of [3] is used defined below without any explanation. (The terms have "left-own-right→new state" structure. An expression (B, R) means "state B or state R", points mean arbitrary states. In the undefined cases the new state must be equal to the old own state.)

$$
\begin{array}{llll}
(B, R) & Q & (Q, N) & \to R \\
(Q, N) & Q & (B, L) & \to L \\
(B, R) & Q & (B, L) & \to N \\
. & R & L & \to N \\
R & L & . & \to N \\
R & (B, N) & (\text{not } L) & \to R \\
. & R & (B, N) & \to N \\
(\text{not } R) & (B, N) & L & \to L \\
(B, N) & L & . & \to N \\
R & (B, N) & L & \to N
\end{array}
$$

For the component S_2 the transition function can be defined by two terms (an expression in parentheses is a 5-bit word, the points mean arbitrary bits):

Term 1. Shift of signals ($a \cdot c \neq 1$ supposed):
$(.b...)(a..e.)(..c.d) \to (abcde)$.

Term 2. Cancellation of the bird mark:
$(.b...)(1..e.)(..1..) \to (0b01e)$.

Area A_3 (rightmost column)

This area serves for reflecting the full-speed signals. The state of the component S_1 is constantly N in each cell (according to [3]), and the transition of the component S_2 is defined by

Term 3: $(.b...)(00.00)(\#) \to (00b00)$.

Area A_1 (leftmost column)

This area requires only 5 states according to the vertical one-dimensional early bird algorithm. These 5 states are chosen from the state-set $S' = S_1 \times S_2$ as follows:

"quiescent" $= (N, 00000)$,
"bird" $= (N, 10000)$,
"right wave" $= (R, 00000)$,
"left wave" $= (R, 10000)$,
"neutral" $= (R, 00100)$.

This choice of states solves two problems:

(i) If in the vertical early bird algorithm a cell is in state "right wave", "left wave" or "neutral", then in the row of this cell all the birds should be cancelled, because they cannot be early birds. Using the above choice of states, a cell in state "right wave", "left wave" or "neutral" shows a state $R(\in S_1)$ for the horizontal early bird algorithm. As a consequence, R states will be generated in the cell-row moving right and cancelling all the birds there.

(ii) It is easy to see that in all cases except (i), a cell in A_1 shows an indifferent state for its right neighbour, because the state $N(\in S_1)$ ensures a correct horizontal early bird algorithm, and the states 10000, 00100 $(\in S_2)$ do not disturb the movement of signals in the cell-row.

Using the above choice of states, the cells in A_1 work with a vertical one-dimensional early bird function, but instead of the spontaneous excitation a "secondary bird generation term" is introduced:

Term 4:
$$\begin{array}{l} (N,00000) \\ \#(N,00000)(.,\ldots.1)\rightarrow(N,10000). \\ (N,00000) \end{array}$$

The solution described above requires $\|S'\|=5\cdot 32=160$ states. Note that the state-set can be reduced to $5\cdot 14=70$ states, but this reduction results in a more complicated transition function therefore its discussion is omitted.

5. The n-dimensional case

Let us consider the n-dimensional cube K defined in point 2. The inner cells of K form an n-dimensional cube K_n of edge length $m-2$: $K_n=\{(i_1, \ldots, i_n)|1\leqq i_j\leqq m-2, j=1, \ldots, n\}$. Using the signals of figure 2, any excitation in K_n can be projected into an $(n-1)$-dimensional cube $K_{n-1}=\{(i_1, \ldots, i_{n-1}, 0)|1\leqq i_j\leqq m-2, j=1, \ldots, n-1\}$, in K_{n-1} secondary excitations are induced. At the same time, in each cell-row of K_n a one-dimensional early bird algorithm is executed to select the local early birds.

For the secondary excitations in K_{n-1} a similar process is used reducing the task to an $(n-2)$-dimensional cube $K_{n-2}=\{(i_1, \ldots, i_{n-2}, 0, 0)|1\leqq i_j\leqq m-2, j=1, \ldots, n-2\}$; similar reduction can be made for K_{n-3}, etc. In $K_1=\{(i_1, 0, \ldots, 0)|1\leqq i_1\leqq m-2\}$ only a one-dimensional early bird algorithm is needed.

For any $i=1, \ldots, n-1$, if a bird in K_i is cancelled then all birds in the corresponding row of K_{i+1} will be cancelled, similarly to the two-dimensional solution. It is not difficult to prove that after a certain time only the global early birds exist in K_n.

Time estimation: Let t be the time-point when the global early birds arise. (Before t the cellular space is in quiescent state.) At time $t+2m$ the signals of the early birds arrive at K_{n-1}, at time $t+4m$ the signals of the secondary birds arrive at K_{n-2}, etc. Thus the earliest excitations appear in K_1 at time $t+(n-1)2m$. The one-dimensional early bird algorithm of [3] requires $3m$ steps, therefore at time $t+(n-1)2m+3m$ all one-dimensional early bird algorithms in $K_1, \ldots, K_n$ are terminated. The cancellation process from K_1 to K_n needs maximum $(n-1)m$ steps, thus at time $t+(n-1)2m+3m+(n-1)m=t+3mn$ all late birds are cancelled in K_n. That is, *the n-dimensional early bird algorithm requires 3mn steps.*

Remark: For the cube K a transition function f is used which is composed of different subfunctions. The subfunctions of the cubes $K_2, \ldots, K_n$ are similar to the function f_2 in point 4, but at the boundary of K_i and K_{i-1} a coding problem arises (see the choosing problem of states in the area A_1). This problem can be solved by increasing the number of states. As a consequence, the state-set S may grow if the dimension degree n is increased.

RESEARCH GROUP ON THEORY OF AUTOMATA
HUNGARIAN ACADEMY OF SCIENCES
SOMOGYI U. 7.
SZEGED, HUNGARY
H-6720

References

[1] ROSENSTIEHL, P., J. R. FIKSEL and A. HOLLIGER, Intelligent graphs: Networks of finite automata capable of solving graph problems, in: Read, R. C. (ed.), *Graph Theory and Computing*, Academic Press, New York, 1972, pp. 219—265.

[2] VOLLMAR, R., On two modified problems of synchronization in cellular automata, *Acta Cybernet.*, v. 3, 1978, pp. 293—300.

[3] LEGENDI, T. and E. KATONA, A 5-state solution of the early bird problem in a one-dimensional cellular space, *Acta Cybernet.*, v. 5, 1981, pp. 173—179.

[4] KLEINE BÜNING, H., The early bird problem is unsolvable in a one-dimensional cellular space with 4 states, *Acta Cybernet.*, v. 6, 1983, pp. 23—31.

[5] PRIESE, L., Über das Early Bird Problem in planaren Zellularräumen, *Informatik-Skripten der Technischen Universität Braunschweig*, Nr. 2, Braunschweig, 1982, pp. 105—110.

[6] LEGENDI, T. and E. KATONA, A solution of the early bird problem in an n-dimensional cellular space (preliminary version), *Informatik-Skripten der Technischen Universität Braunschweig*, Nr. 2, Braunschweig, 1982, pp. 60—64.

[7] VOLLMAR, R., *Algorithmen in Zellularautomaten*, B. G. Teubner, Stuttgart, 1979.

(Received Febr. 14, 1984)