On α_i-product of tree automata

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In the theory of finite automata it is a central problem to represent a given automaton by composition of — possibly simpler — automata. The composition of tree automata has received little attention. Namely, the cascade product of tree automata was studied in [4] and the work [5] contains the investigation of the general product of tree automata (see also [1]). In this paper generalizing the notion of α_i-product (cf. [2]), we introduce the α_i-product of tree automata, and using the idea in [3] give necessary and sufficient conditions for a system of tree automata to be isomorphically complete with respect to the α_i-product. From the characterizations of complete systems we obtain the α_i-products constitute a proper hierarchy.

1. Definitions

By a *set of operational symbols* we mean the nonempty union $\Sigma=\Sigma_0\cup\Sigma_1\cup\ldots$ of pairwise disjoint sets of symbols, and for any nonnegative integer m, Σ_m is called the *set of m-ary operational symbols*. It is said that the *rank* or *arity* of a symbol $\sigma\in\Sigma$ is m if $\sigma\in\Sigma_m$. Now let a set Σ of operational symbols be given. A set R of nonnegative integers is called the *rank-type of* Σ if for any m, $\Sigma_m\neq\emptyset$ if and only if $m\in R$. Next we shall work always under a fixed rank-type R.

Let Σ be a set of operational symbols with rank-type R. Then by a *Σ-algebra* $\mathcal{A}$ we mean a pair consisting of a nonempty set A (of elements of $\mathcal{A}$) and a mapping that assigns to every operational symbol $\sigma\in\Sigma$ an m-ary operation $\sigma^{\mathcal{A}}\colon A^m\to A$, where the arity of σ is m. The operation $\sigma^{\mathcal{A}}$ is called the *realization of σ in $\mathcal{A}$*. The mapping $\sigma\to\sigma^{\mathcal{A}}$ will not be mentioned explicitly, but we write $\mathcal{A}=(A,\Sigma)$. The Σ-algebra $\mathcal{A}$ is *finite* if A is finite, and it is of *finite type* if Σ is finite. By a *tree automaton* we mean a finite algebra of finite type. We say that the rank-type of a tree automaton $\mathcal{A}=(A,\Sigma)$ is R if the rank-type of Σ is R. Let us denote by $\mathfrak{A}_R$ the class of all tree automata with rank-type R.

Now let i be a fixed nonnegative integer, and let

$$\mathcal{A}=(A,\Sigma)\in\mathfrak{A}_R,\quad \mathcal{A}_j=(A_j,\Sigma^j)\in\mathfrak{A}_R\quad (j\doteq 1,\ldots,k).$$

Moreover, take a family ψ of mappings

$$\psi_{mj}\colon (A_1\times\ldots\times A_k)^m\times\Sigma_m\to\Sigma^j_m,\quad m\in R,\ 1\leqq j\leqq k.$$

It is said that the tree automaton $\mathscr{A}$ is the α_i-product of $\mathscr{A}_j$ $(j=1, \ldots, k)$ with respect to ψ if the following conditions are satisfied:

(1) $A = \prod_{t=1}^{k} A_t$,

(2) for any $m \in R$, $j \in \{1, \ldots, k\}$,

$$((a_{11}, \ldots, a_{1k}), \ldots, (a_{m1}, \ldots, a_{mk})) \in (A_1 \times \ldots \times A_k)^m$$

the mapping ψ_{mj} is independent of elements a_{rs} $(1 \leqq r \leqq m, j+i \leqq s)$,

(3) for any $m \in R$, $\sigma \in \Sigma_m$, $((a_{11}, \ldots, a_{1k}), \ldots, (a_{m1}, \ldots, a_{mk})) \in (A_1 \times \ldots \times A_k)^m$,

$$\sigma^{\mathscr{A}}((a_{11}, \ldots, a_{1k}), \ldots, (a_{m1}, \ldots, a_{mk})) = (\sigma_1^{\mathscr{A}_1}(a_{11}, \ldots, a_{m1}), \ldots, \sigma_k^{\mathscr{A}_k}(a_{1k}, \ldots, a_{mk})),$$

where

$$\sigma_j = \psi_{mj}((a_{11}, \ldots, a_{1k}), \ldots, (a_{m1}, \ldots, a_{mk}), \sigma) \quad (j = 1, \ldots, k).$$

For the above product we shall use the notation $\prod_{j=1}^{k} \mathscr{A}_j(\Sigma, \psi)$ and sometimes we shall write only those variables of ψ_{mj} on which ψ_{mj} depends.

Finally, we shall denote by $[\sqrt[t]{n}]$ the largest integer less than or equal to $\sqrt[t]{n}$.

2. Completeness

Let i be a fixed nonnegative integer and $\mathfrak{B} \subseteq \mathfrak{A}_R$. $\mathfrak{B}$ is called *isomorphically complete* for $\mathfrak{A}_R$ with respect to the α_i-product if any tree automaton from $\mathfrak{A}_R$ can be embedded isomorphically into an α_i-product of tree automata from $\mathfrak{B}$. Furthermore, $\mathfrak{B}$ is called *minimal isomorphically complete* system if $\mathfrak{B}$ is isomorphically complete and for arbitrary $\mathscr{A} \in \mathfrak{B}$, $\mathfrak{B} \setminus \{\mathscr{A}\}$ is not isomorphically complete.

For any natural number $n>0$ let us denote by $\mathscr{B}_n = (\{0, \ldots, n-1\}, \theta^n)$ the tree automaton where for every m-ary operation ϱ: $\{0, \ldots, n-1\}^m \to \{0, \ldots, n-1\}$ there exists exactly one $\sigma \in \theta_m^n$ with $\sigma^{\mathscr{B}_n} = \varrho$ provided that $m \in R$.

The following statement is obvious.

Lemma. *If* $\mathscr{A}_j \in \mathfrak{A}_R$ $(j=1, 2, 3)$ and $\mathscr{A}_j$ can be embedded isomorphically into and α_i-product of $\mathscr{A}_{j+1}$ with a single factor $(j=1, 2)$ then $\mathscr{A}_1$ can be embedded isomorphically into an α_i-product of $\mathscr{A}_3$ with a single factor.

First we consider the special case $R=\{0\}$. Then the following statement is obvious.

Theorem 1. $\mathfrak{B} \subseteq \mathfrak{A}_R$ is isomorphically complete for $\mathfrak{A}_R$ with respect to the α_i-product if and only if there exists an $\mathscr{A} \in \mathfrak{B}$ such that $\mathscr{B}_2$ can be embedded isomorphically into an α_i-product of $\mathscr{A}$ with a single factor.

Now let us suppose $R \neq \{0\}$. Then the results of completeness is based on the following Theorem.

Theorem 2. If the tree automaton $\mathscr{B}_n$ $(n>1)$ can be embedded isomorphically into an α_i-product $\prod_{j=1}^{k} \mathscr{A}_j(\theta^n, \psi)$ of the tree automata $\mathscr{A}_j \in \mathfrak{A}_R$ $(j=1, ..., k)$ then $\mathscr{B}_{[i^* \sqrt{n}]}$ can be embedded isomorphically into an α_i-product of $\mathscr{A}_j$ with a single factor for some $j \in \{1, ..., k\}$, where $i^*=i$ if $i>0$ and $i^*=1$ else.

Proof. If $k=1$ then the statement is obvious. Now let $k>1$. Assume that $\mathscr{B}_n$ can be embedded isomorphically into the α_i-procut $\mathscr{A} = \prod_{j=1}^{k} \mathscr{A}_j(\theta^n, \psi)$ and let μ denote a suitable isomorphism. Let $\mu(t)=(a_{t1}, ..., a_{tk})$ $(t=0, ..., n-1)$. We may suppose that there exist natural numbers $u \neq v$ $(0 \leqq u,\ v \leqq n-1)$ such that $a_{u1} \neq a_{v1}$ since otherwise $\mathscr{B}_n$ can be embedded isomorphically into an α_i-product of $\mathscr{A}_j$ $(j=2, ..., k)$. Now assume that there exist natural numbers $p \neq q$ $(0 \leqq p, q \leqq n-1)$ with $a_{ps}=a_{qs}$ $(s=1, ..., i^*)$. For any t $(0 \leqq t \leqq n-1)$ let us denote by $\sigma_{pt}^{\mathscr{B}_n}$ the m-ary operation of $\mathscr{B}_n$ for which $\sigma_{pt}^{\mathscr{B}_n}(0, ..., 0, p)=t$ and $\sigma_{pt}^{\mathscr{B}_n}(0, ..., 0, q)=q$, for some $m \in R$. Such operations exist since $R \neq \{0\}$. Then for any $t \in \{0, ..., n-1\}$

$$(a_{t1}, ..., a_{tk}) = \mu(t) = \mu(\sigma_{pt}^{\mathscr{B}_n}(0, ..., 0, p)) = \sigma_{pt}^{\mathscr{A}}(\mu(0), ..., \mu(0), \mu(p)) =$$
$$= (\sigma_1^{\mathscr{A}_1}(a_{01}, ..., a_{01}, a_{p1}), \sigma_2^{\mathscr{A}_2}(a_{02}, ..., a_{02}, a_{p2}), ..., \sigma_k^{\mathscr{A}_k}(a_{0k}, ..., a_{0k}, a_{pk}))$$

holds, and so $a_{t1}=\sigma_1^{\mathscr{A}_1}(a_{01}, ..., a_{01}, a_{p1})$ where

$$\sigma_1 = \psi_{m1}((a_{01}, ..., a_{0k}), ..., (a_{01}, ..., a_{0k}), (a_{p1}, ..., a_{pk}), \sigma_{pt}) =$$
$$= \psi_{m1}(a_{01}, ..., a_{0i^*}, a_{p1}, ..., a_{pi^*}, \sigma_{pt}) \quad \text{if} \quad i > 0$$

and $\sigma_1=\psi_{m1}(\sigma_{pt})$ if $i=0$. In the same way we obtain the equality

$$a_{q1} = \bar{\sigma}_1^{\mathscr{A}_1}(a_{01}, ..., a_{01}, a_{q1})$$

where

$$\bar{\sigma}_1 = \psi_{m1}(a_{01}, ..., a_{0i^*}, a_{q1}, ..., a_{qi^*}, \sigma_{pt}) \quad \text{if} \quad i > 0$$

and

$$\bar{\sigma}_1 = \psi_{m1}(\sigma_{pt}) \quad \text{if} \quad i = 0.$$

Since $a_{ps}=a_{qs}$ $(s=1, ..., i^*)$ we obtain that $\sigma_1=\bar{\sigma}_1$ which implies the equality $a_{t1}=a_{q1}$ for any $t \in \{0, ..., n-1\}$. This contradicts our assumption $a_{u1} \neq a_{v1}$, therefore the elements $(a_{t1}, ..., a_{ti^*})$ $(0 \leqq t \leqq n-1)$ are pairwise different. Now we shall show that in this case $\mathscr{B}_n$ can be embedded isomorphically into an α_i-product $\bar{\mathscr{A}} = \prod_{j=1}^{i^*} \mathscr{A}_j(\theta^n, \varphi)$. Indeed, let us define the family φ of mappings as follows: for any $m \in R$, $j \in \{1, ..., i^*\}$, $((a_1^1, ..., a_1^{i^*}), ..., (a_m^1, ..., a_m^{i^*})) \in \prod_{j=1}^{i^*} A_j$, $\sigma \in \theta^n$ elements

(1) if $i>0$ then

$$\varphi_{mj}((a_1^1, ..., a_1^{i^*}), ..., (a_m^1, ..., a_m^{i^*}), \sigma) = \begin{cases} \psi_{mj}((a_{u_1 1}, ..., a_{u_1 k}), ..., (a_{u_m 1}, ..., a_{u_m k}), \sigma) \\ \text{if there exist } u_1, ..., u_m \in \{0, ..., n-1\} \\ \text{such that } a_s^t = a_{u_s t} (t = 1, ..., i^*, s=1, ..., m), \\ \text{arbitrary operational symbol from} \\ \Sigma_m^j \text{ otherwise,} \end{cases}$$

(2) if $i=0$ then $\varphi_{mj}(\sigma)=\psi_{mj}(\sigma)$.

It is clear that φ_{mj} is well defined. On the other hand, it is easy to see that the mapping $\nu(t)=(a_{t1}, ..., a_{ti^*})$ $(t=0, ..., n-1)$ is an isomorphism of $\mathcal{B}_n$ into $\bar{\mathcal{A}}$. Using this isomorphism ν we prove that $\mathcal{B}_{[i^*\sqrt{n}]}$ can be embedded isomorphically into an α_i-product of $\mathcal{A}_j$ with a single factor for some $j\in\{1, ..., i^*\}$. If $i=0$ or $i=1$ then this statement obviously holds. Now assume that $i>1$. Since the elements $(a_{t1}, ..., a_{ti^*})$ $(t=0, ..., n-1)$ are pairwise different, there exists an $s\in\{1, ..., i^*\}$ such that the number of pairwise different elements among $a_{0s}, a_{1s}, ..., a_{n-1s}$ is greater than or equal to $v=[i^*\sqrt{n}]$. Without loos of generality we may assume that $a_{0s}, ..., a_{v-1s}$ are pairwise different elements of $\mathcal{A}_s$. For any $m\in R$, $\sigma\in\theta_m^v$ let us denote by $\bar{\sigma}$ an operational symbol from θ_m^n for which $\sigma^{\mathcal{B}_n}|_{\{0, ..., v-1\}^m}=\sigma^{\mathcal{B}_v}$. Now let us define the α_i-product $\mathcal{A}_s(\theta^v, \bar{\varphi})$ as follows: for any $m\in R$, $\sigma\in\theta_m^v$, $(a_{u_1s}, ..., a_{u_ms})\in A_s^m$

$$\bar{\varphi}_m(a_{u_1s}, ..., a_{u_ms}, \sigma) = \begin{cases} \varphi_{ms}\big((a_{u_11}, ..., a_{u_1i^*}), ..., (a_{u_m1}, ..., a_{u_mi^*}), \bar{\sigma}\big) \text{ if} \\ 0 \leqq u_t \leqq v-1 \; (t = 1, ..., m), \\ \text{arbitrary operational symbol from } \Sigma_m^s \text{ otherwise.} \end{cases}$$

It can be easily see that the correspondence ν': $t\to a_{ts}$ $(t=0, ..., v-1)$ is an isomorphism of $\mathcal{B}_v$ into $\mathcal{A}_s(\theta^v, \bar{\varphi})$, which completes the proof of Theorem 2.

Theorem 3. $\mathfrak{B}\subseteq\mathfrak{A}_R$ is isomorphically complete for $\mathfrak{A}_R$ with respect to the α_0-product if and only if for any natural number $n>1$ there exists an $\mathcal{A}\in\mathfrak{B}$ such that $\mathcal{B}_n$ can be embedded isomorphically into an α_0-product of $\mathcal{A}$ with a single factor.

Proof. The necessity follows from Theorem 2. To prove the sufficiency let us observe that any tree automaton $\mathcal{A}\in\mathfrak{A}_R$ with $|A|=n$ can be embedded isomorphically into an α_0-product of $\mathcal{B}_n$ with a single factor. From this fact, by our Lemma, we obtain the completeness of $\mathfrak{B}$.

Now let $i>0$ be a fixed nonnegative integer. Then in a similar way as above we obtain the following result.

Theorem 4. $\mathfrak{B}\subseteq\mathfrak{A}_R$ is isomorphically complete for $\mathfrak{A}_R$ with respect to the α_i-product if and only if for any natural number $n>1$ there exists an $\mathcal{A}\in\mathfrak{B}$ such that $\mathcal{B}_n$ can be embedded isomorphically into an α_i-product of $\mathcal{A}$ with a single factor.

Since an α_i-product with a single factor is an α_1-product with a single factor, by Theorem 4, we get the next corollary.

Corollary 1. $\mathfrak{B}\subseteq\mathfrak{A}_R$ is isomorphically complete for $\mathfrak{A}_R$ with respect to the α_1-product if and only if $\mathfrak{B}$ is isomorphically complete for $\mathfrak{A}_R$ with respect to the α_i-product.

Now let i be a nonnegative integer. Then we have the following result for the minimal isomorphically complete systems in the case $R\neq\{0\}$.

Theorem 5. There exists no system $\mathfrak{B}\subseteq\mathfrak{A}_R$ which is isomorphically complete for $\mathfrak{A}_R$ with respect to the α_i-product and minimal.

Proof. Let $\mathfrak{B} \subseteq \mathfrak{A}_R$ be isomorphically complete for $\mathfrak{A}_R$ with respect to the α_i-product. Moreover, let $\mathscr{A} \in \mathfrak{B}$ with $|A| = n$. It is obvious that $\mathscr{A}$ can be embedded isomorphically into an α_i-product of $\mathscr{B}_s$ with a single factor if $s \geqq n$. Take a natural number $s > n$. By Theorem 3 and Theorem 4, there exists an $\bar{\mathscr{A}} \in \mathfrak{B}$ such that $\mathscr{B}_s$ can be embedded isomorphically into an α_i-product of $\bar{\mathscr{A}}$ with a single factor. Therefore, by our Lemma, $\mathscr{A}$ can be embedded isomorphically into an α_i-product of $\bar{\mathscr{A}}$ with a single factor. From this it follows that $\mathfrak{B} \setminus \{\mathscr{A}\}$ is isomorphically complete for $\mathfrak{A}_R$ with respect to the α_i-product, showing that $\mathfrak{B}$ is not minimal.

3. The hierarchy of α_i-products

Let $R \neq \{0\}$ be a fixed rank-type. Take a nonempty set $M \subseteq \mathfrak{A}_R$, and let i be an arbitrary nonnegative integer. Let $\alpha_i(M)$ denote the class of all tree automata from $\mathfrak{A}_R$ which can be embedded isomorphically into an α_i-product of tree automata from M. It is said that the α_i-product is *isomorphically more general* than the α_j-product if for any set $M \subseteq \mathfrak{A}_R$ the relation $\alpha_j(M) \subseteq \alpha_i(M)$ holds and there exists at least one set $\bar{M} \subseteq \mathfrak{A}_R$ such that $\alpha_j(\bar{M})$ is a proper subclass of $\alpha_i(\bar{M})$. This notion was introduced in [2].

As far as the hierarchy of the α_i-products is concerned, we have the following Theorem.

Theorem 6. For any i, j $(i, j \in \{0, 1, \ldots\})$ the α_i-product is isomorphically more general than the α_j product if $j < i$.

Proof. We shall prove that the α_1-product is isomorphically more general than the α_0-product and the α_{i+1}-product is isomorphically more general than the α_i-product if $i \geqq 1$.

First let $M = \{\mathscr{A}_2\}$, where $\mathscr{A}_2 = (\{1, 2\}, \bigcup_{m \in R} \{\sigma_{m1}, \sigma_{m2}\})$ and the operations of $\mathscr{A}_2$ are defined as follows: for any $0 \neq m$, $m \in R$, $(a_1, \ldots, a_m) \in \{1, 2\}^m$

$$\sigma_{m1}^{\mathscr{A}_2}(a_1, \ldots, a_m) = \begin{cases} 1 & \text{if} \quad a_m = 2, \\ 2 & \text{if} \quad a_m = 1, \end{cases}$$

$$\sigma_{m2}^{\mathscr{A}_2}(a_1, \ldots, a_m) = a_m,$$

and $\sigma_{01}^{\mathscr{A}_2} = 1$, $\sigma_{02}^{\mathscr{A}_2} = 2$ if $0 \in R$.

Now let us denote by $\mathscr{A}_3 = (\{1, 2, 3\}, \Sigma')$ the tree automaton where for any $0 \neq m \in R$ $\sigma \in \Sigma'_m$, $(a_1, \ldots, a_m) \in \{1, 2, 3\}^m$

$$\sigma^{\mathscr{A}_3}(a_1, \ldots, a_m) = \begin{cases} a_m + 1 & \text{if} \quad a_m < 3, \\ 3 & \text{if} \quad a_m = 3, \end{cases}$$

and $\bar{\sigma}^{\mathscr{A}_3} = 1$ if $0 \in R$ and $\bar{\sigma} \in \Sigma'_0$.

It is easy to see that $\mathscr{A}_3 \notin \alpha_0(M)$ and $\mathscr{A}_3 \in \alpha_1(M)$ which yields the required inclusion $\alpha_0(M) \subset \alpha_1(M)$.

Now let $i \geqq 1$ and $M = \{\mathcal{B}_2\}$. Then, by the proof of Theorem 2, we obtain that $\mathcal{B}_{2^i+1} \notin \alpha_i(M)$. On the other hand, we shall show that $\mathcal{B}_{2^i+1} \in \alpha_{i+1}(M)$ which yields the required inclusion $\alpha_i(M) \subset \alpha_{i+1}(M)$. To prove the above statement it is enough to show that $\mathcal{B}_{2^i} \in \alpha_i(M)$ if $i > 1$. Indeed, let us take the α_i-product $\mathcal{A} = \prod_{j=1}^{i} \mathcal{B}_2(\theta^{2^i}, \psi)$ where the family ψ of mappings is defined as follows: for any $0 \neq m$, $\sigma \in \theta_m^{2^i}$,

$$((a_{11}, \ldots, a_{1i}), \ldots, (a_{m1}, \ldots, a_{mi})) \in (\{0, 1\}^i)^m$$

if

$$\sigma^{\mathcal{B}_2 i}\Big(\sum_{t=1}^{i} a_{1t} 2^{i-t}, \ldots, \sum_{t=1}^{i} a_{mt} 2^{i-t}\Big) = w = \sum_{t=1}^{i} a_{wt} 2^{i-t} \quad \text{and} \quad \bar{\sigma}^{\mathcal{B}_2}(a_{1j}, \ldots, a_{mj}) = a_{wj}$$

then

$$\psi_{mj}((a_{11}, \ldots, a_{1i}), \ldots, (a_{m1}, \ldots, a_{mi}), \sigma) = \bar{\sigma}.$$

In the case $\sigma \in \theta_0^{2^i}$ if $\sigma^{\mathcal{B}_2 i} = \sum_{t=1}^{i} a_{vt} \cdot 2^{i-t}$ and $\bar{\sigma}^{\mathcal{B}_2} = a_{vj}$ then $\psi_{mj}(\sigma) = \bar{\sigma}$.

It is easy to see that $\mathcal{B}_{2^i}$ can be embedded isomorphically into $\mathcal{A}$ under the isomorphism μ defined as follows: if $w = \sum_{t=1}^{i} a_t 2^{i-t}$ then $\mu(w) = (a_1, \ldots, a_i)$ $(w = 0, \ldots, 2^i - 1)$.

4. A decidability result

In this section we show that it is decidable if an algebra can be represented isomorphically by an α_i-product of algebras from a given finite set.

Theorem 7. For any nonnegative integer i, $\mathcal{A} \in \mathfrak{A}_R$ and finite set $M \subseteq \mathfrak{A}_R$ it can be decided whether or not $\mathcal{A} \in \alpha_i(M)$.

Proof. Let us suppose that $\mathcal{A}$ with $A = \{a_1, \ldots, a_k\}$ can be embedded isomorphically into an α_i-product $\mathcal{B} = \prod_{j=1}^{s} \mathcal{A}_j(\Sigma, \varphi)$ of tree automata from M. Let $V = \max\{|A_t| : \mathcal{A}_t \in M\}$ and let $(a_{u1}, \ldots, a_{us})$ denote the image of a_u under a suitable isomorphism μ $(u = 1, \ldots, k)$. We define an equivalence relation π on the set of indices of the α_i-product $\mathcal{B}$ as follows: for any l, n $(1 \leqq l, n \leqq s)$, $l \pi n$ holds if and only if $\mathcal{A}_l = \mathcal{A}_n$ and $a_{tl} = a_{tn}$ for all $t = 1, \ldots, k$.

It is easy to see that the partition corresponding to π has at most $|M| \cdot V^k$ blocks. Since $\mu(A)$ is a subalgebra of $\mathcal{B}$, if $a_{tl} = a_{tn}$ $(t = 1, \ldots, k)$ then the l-th and n-th components of $\mu(\sigma(a^1, \ldots, a^m))$ are equal, where $m \in R$, $\sigma \in \Sigma_m$, $a^j \in A$ $(j = 1, \ldots, m)$. From this it follows that $\mathcal{A}$ can be embedded isomorphically into an α_i-product of tree automata from M with at most $|M| \cdot V^k$ factors.

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