Evaluated grammars

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1. Introduction

Mechanisms which regulate the application of the rules belong to the most important devices in order to enlarge the generative capacity of context free grammars. A common idea is that not every derivation leading from the start symbol to a terminal word is acceptable, but there is a control device which lets through acceptable derivations only. For instance, an application of some production determines which productions are applicable in the next step (this is called a programmed grammar), or some productions can never be applied if any other applicable (an ordered grammar). In a matrix grammar one has to apply only certain previously specified strings of productions or, more generally, the string of productions corresponding to a derivation must belong to a set of previously specified strings (a grammar with a control set) — see [3].

In this paper, the notion of evaluated grammar is introduced. The derivation process in this generative mechanism is regulated by a certain evaluation of some symbols occurring in sentential forms.

We believe that the introduction of the new type of grammar with a restriction in derivation introduced here is very useful because of three reasons:

(i) evaluated grammars represent a simple and very natural extension of context-free grammars;

(ii) evaluated grammars are considerably more powerful than context-free grammars;

(iii) some classes of languages generated by parallel rewriting systems (e.g. EOL languages) can be characterized by evaluated grammars in a natural way.

2. Preliminaries

We introduce here only briefly the notions needed in this paper. For a more detailed discussion, as well as for background material and motivation, the reader is referred to [2, 3].

Let α be a word over an alphabet Σ. The alphabet of α, alph α, is the set of all symbols (from Σ) that appear at least once in α.

A context free grammar is quadruple $G=(\Sigma, P, S, \Delta)$ where, as usual, Σ is a finite alphabet, $\Delta \subseteq \Sigma$ is the terminal alphabet and $\Sigma \setminus \Delta$ is the nonterminal alphabet, $P \subseteq (\Sigma \setminus \Delta) \times \Sigma^*$ is a finite set of productions, where a production (A, α) is usually written as $A \to \alpha$, and S in $\Sigma \setminus \Delta$ is the start symbol. For arbitrary words $x, y \in \Sigma^*$ and production $A \to \alpha$ we write $xAy \Rightarrow x\alpha y$, and denote the reflexive, transitive closure of $\Rightarrow$ by $\Rightarrow^*$. The language generated by G, denoted $L(G)$, is defined by $L(G)=\{x \in \Delta^*: S \Rightarrow^* x\}$.

A context free grammar $G=(\Sigma, P, S, \Delta)$ is called regular if every production $A \to \alpha$ from P satisfies $\alpha \in \Delta(\Sigma \setminus \Delta) \cup \Delta$.

An ETOL system G consits of $m+3 (m \geqq 1)$ components $G=(\Sigma, P_1, \ldots, P_m, S, \Delta)$ where Σ, Δ, S are defined identically as for context free grammars, and where every P_i is a finite subset of $\Sigma \times \Sigma^*$ such that for every $a \in \Sigma$ at least one pair (a, α) occurs in P_i. The pairs in P_i are again called productions and usually written as $a \to \alpha$. For an arbitrary word $x=a_1 a_2 \ldots a_k$, $a_i \in \Sigma$, and productions $a_1 \to \alpha_1, \ldots, a_k \to \alpha_k$ of the same set P_j we write $a_1 a_2 \ldots a_k \Rightarrow \alpha_1 \alpha_2 \ldots \alpha_k$ and denote the reflexive, transitive closure of $\Rightarrow$ by $\Rightarrow^*$. The language generated by G, $L(G)$, is defined by $L(G)= =\{x \in \Delta^*: S \Rightarrow^* x\}$. An ETOL system with a single set of productions is called EOL system.

For $n>0$, an n-parallel right linear grammar (see [1]) is a quintuple $G=(\Sigma, P, S, \Delta, n)$ where Σ, Δ, S are defined identically as for context free grammars, and $P \subseteq (\Sigma \setminus (\Delta \cup \{S\})) \times (\Delta^*(\Sigma \setminus (\Delta \cup \{S\})) \cup \Delta^+) \cup \{S\} \times (\Delta^* \cup (\Sigma \setminus (\Delta \cup \{S\}))^n)$ is a finite set of rules, where a production (A, α) is usually written as $A \to \alpha$. The yield relation is defined as follows: for $x, y \in \Sigma^*$, $x \Rightarrow y$ if and only if either $x=S$ and $S \to y \in P$ or $x=y_1 X_1 \ldots y_n X_n$ and $y=y_1 x_1 \ldots y_n x_n$, where $y_i \in \Delta^*$, $x_i \in \in \Delta^*(\Sigma \setminus (\{S\} \cup \Delta)) \cup \Delta^+$, $X_i \in \Sigma \setminus \Delta$ and $X_i \to x_i \in P$, $1 \leqq i \leqq n$. The relation $\Rightarrow$ can be extended to give $\Rightarrow^*$ as above. The notion of the language generated by G can be introduced just as for context free grammars. (An n-parallel right linear grammar G is in normal form if

$$G = (\Delta \cup K_1 \cup \ldots \cup K_n \cup \{S\}, P, S, \Delta, n),$$

S is not in $\Delta \cup K_1 \cup \ldots \cup K_n$, K_i are mutually disjoint nonterminal sets, if $S \to X_1 \ldots X_n \in P$ and $X_1 \ldots X_n \in (K_1 \cup \ldots \cup K_n)^*$ then $X_i \in K_i$, $1 \leqq i \leqq n$, and if $X_i \to y Y_j \in P$, $X_i \in K_i$ and $Y_j \in K_j$ then $i=j$.)

The families of languages generated by context free, regular and n-parallel right linear grammars are denoted by $\mathscr{L}(\mathrm{CF})$, $\mathscr{L}(\mathrm{RG})$ and $\mathscr{L}(n-\mathrm{PRL})$, respectively. Let $\mathscr{L}(\mathrm{PRL})=\bigcup_{i=1}^{\infty} \mathscr{L}(i-\mathrm{PRL})$. Families of languages generated by ETOL systems, ETOL systems of finite index (see [2]) and EOL systems are denoted by $\mathscr{L}(\mathrm{ETOL})$, $\mathscr{L}_{\mathrm{FIN}}(\mathrm{ETOL})$ and $\mathscr{L}(\mathrm{EOL})$, respectively.

3. Definition of evaluated grammars

Intuitively, an evaluated grammar is very much like a context free grammar. However, some symbols (including terminals) in a given sentential form of an evaluated grammar can have a certain value associated (a non-negative integer). In one derivation step either a nonterminal without associated value in a usual "context free" way or a nonterminal with the least value that occurs in the given sentential form is rewritten. In the latter case the nonterminal is rewritten again in a usual way but, in addition new evaluation is assigned to apriori specified (on the right side of the rule applied) symbols.

Formally, let N be the set of non-negative integers and let Σ be an alphabet. We denote members of $V=\Sigma\times N$ by $a_{(i)}$ where a is in Σ and i is in N, then in a natural way we can define V^*. Define the letter-to-letter homomorphism $v\colon (V\cup\Sigma)^*\to\Sigma^*$ by $v(a_{(i)})=a$ for all $a_{(i)}$ in V and $v(a)=a$ for all $a\in\Sigma$. An evaluated grammar, EG, is a construct $G=(\Sigma, P, S, \Delta)$ where $\Delta\subseteqq\Sigma$ is a terminal alphabet, $P\subseteqq(\Sigma\setminus\Delta)\times\times(V\cup\Sigma)^*$ is a finite set of productions, where $(A,\alpha)\in P$ is usually written as $A\to\alpha$, and $S\in\Sigma\setminus\Delta$ is a start symbol. For all $\alpha,\beta\in(V\cup\Sigma)^*$ we write $\alpha\underset{G}{\Rightarrow}\beta$ (or simply $\alpha\Rightarrow\beta$ if G is understood) if $\beta=\alpha_1\gamma\alpha_2$ for some $\alpha_1,\alpha_2\in(V\cup\Sigma)^*$ and either $\gamma\in\Sigma^*$, $\alpha=\alpha_1 A\alpha_2$ and $A\to\gamma\in P$ or $\alpha=\alpha_1 A_{(i)}\alpha_2$ for some $A_{(i)}\in V$,

$$\gamma=\delta_0 B_{1_{(i+k_1)}}\delta_1\dots B_{n_{(i+k_n)}}\delta_n, A\to\delta_0 B_{1_{(k_1)}}\delta_1\dots B_{n_{(k_n)}}\delta_n\in P$$

where $\delta_j\in\Sigma^*$, $B_{j_{(k)_j}}$, $B_{j_{(i+k_j)}}\in V$ (i.e. $B_j\in\Sigma$, i, $k_j\in N$), $0\leqq j\leqq n$ for some $n\geqq 0$ (where $n=0$ implies $\gamma=\delta_0$ and $A\to\delta_0\in P$) and $i\leqq m$ for every $X_{(m)}\in\operatorname{alph}\alpha_1\alpha_2\cap V$. The language generated by G, $L(G)$, is defined by $L(G)=\{v(x)\colon v(x)\in\Delta^*$ and $S_{(0)}\Rightarrow^* x\}$ where $\Rightarrow^*$ is the transitive,reflexive closure of $\Rightarrow$.

Now we introduce some special cases of evaluated grammars. Let $G=(\Sigma, P\ S, \Delta)$ be an EG and let n be a positive integer. We say that G is n-regular if it has the following properties:

(1) if $S\to\alpha\in P$ then $v(\alpha)=X_1\dots X_n$
with $X_i\in\Sigma\setminus(\Delta\cup\{S\})$, $1\leqq i\leqq n$;

(2) if $A\to\alpha\in P$ and $A\neq S$ then
$v(\alpha)\in\Delta(\Sigma\setminus(\Delta\cup\{S\}))\cup\Delta$.

We say that the EG G is regular, RGEG, if it is n-regular for some positive integer n.

We say that an EG G is binary, BEG, if

$$P\subseteqq(\Sigma\setminus\Delta)\times((\Delta\times\{0\})\cup((\Sigma\setminus\Delta)\times\{1\})\cup\Delta)^*.$$

A binary regular EG (i.e. an EG which is as regular, as binary) will be denoted by BRGEG.

We use $\mathscr{L}$(E), $\mathscr{L}$(RGE), $\mathscr{L}$(BE) and $\mathscr{L}$(BRGE) to denote the families of languages generated by evaluated, regular evaluated, binary evaluated and binary regular evaluated grammars, respectively.

4. Examples

We now consider some examples to give insight into evaluated grammars. We usually define evaluated grammars by simply listing their productions in Backus-Naur form. In this case we use S to denote the start symbol, early upper case Roman letters to denote nonterminals and early lower case Roman letters to denote terminals.

Example 1. Let

$$G_1\colon\ S \to A_{(1)}B_{(1)}C_{(1)};\quad A \to aA_{(1)}|a_{(0)};$$

$$B \to bB_{(1)}|b_{(0)};\qquad C \to cC_{(1)}|c_{(0)}$$

be a BRGEG. Then, e.g., for the word $aabbcc$ there exists a derivation in G:

$$S_{(0)} \Rightarrow A_{(1)}B_{(1)}C_{(1)} \Rightarrow aA_{(2)}B_{(1)}C_{(1)} \Rightarrow aA_{(2)}bB_{(2)}C_{(1)} \Rightarrow$$

$$\Rightarrow aA_{(2)}bB_{(2)}cC_{(2)} \Rightarrow aA_{(2)}bb_{(2)}cC_{(2)} \Rightarrow$$

$$\Rightarrow aA_{(2)}bb_{(2)}cc_{(2)} \Rightarrow aa_{(2)}bb_{(2)}cc_{(2)}$$

and thus we get

$$v(aa_{(2)}bb_{(2)}cc_{(2)}) = aa\,bb\,cc.$$

Clearly, G generates a well-known context-sensitive language:

$$L(G_1) = \{a^n b^n c^n\colon\ n \geqq 1\}.$$

Example 2. Consider the RGEG

$$G_2\colon\ S \to A_{(0)}B_{(1)}A_{(0)};\quad A \to aA_{(2)}|a_{(1)};$$

$$B \to bB_{(2)}|b_{(0)}C\,|b_{(0)};\quad C \to bc|b.$$

The reader can easily check that

$$L(G_2) = \{a^k b^l a^k\colon\ l \geqq k \geqq 1\}.$$

It is well-known (see [2]) that $L(G_2)$ is not an EOL language.

Example 3. Let

$$G_3\colon\ S \to S_{(1)}S_{(1)}|a_{(0)}$$

$$L(G_3) = \{a^{2^n}\colon\ n \geqq 1\}$$

be a BEG. It is not difficult to show that which is not an ETOL language of finite index (see [2]).

5. Generating power of evaluated grammars

From the definition of an EG it is easy to see that every context free language can be obtained as the language of some EG. Moreover, from examples in the previous section it follows that the class of context free languages is properly contained in $\mathscr{L}$(E). The purpose of this section is to show that $\mathscr{L}$(E) is included in $\mathscr{L}$(ETOL).

Theorem 1. $\mathscr{L}(\mathrm{E}) \subseteq \mathscr{L}(\mathrm{ETOL})$.

Proof. Let

$$G = (\Sigma, P, S, \Delta)$$

be an EG and let k be an arbitrary but fixed non-negative integer such that for every production $A \to \alpha_1 B_{(i)} \alpha_2 \in P$, where $A \in \Sigma \setminus \Delta_1$, $\alpha_1\alpha_2 \in (V \cup \Sigma)^*$, $B_{(i)} \in V$, it holds that $i \leqq k$. Consider a new alphabet $\bar{\Sigma} = \{[A, i]: A \in \Sigma \text{ and } 0 \leqq i \leqq k\}$ and let F be a new "block" symbol. Let $\bar{\Delta} = \{[a, i]: a \in \Delta \text{ and } 0 \leqq i \leqq k\}$; clearly $\bar{\Delta} \subseteqq \bar{\Sigma}$. Now we define four new tables of productions P_i, $1 \leqq i \leqq 4$, as follows:

$$P_1 = \{[A, 0] \to x_0 [B_1, k_1] x_1 \dots [B_n, k_n] x_n:$$

$$A \to x_0 B_{1(k_1)} x_1 \dots B_{n(k_n)} x_n \in P,$$

$$A \in \Sigma \setminus \Delta, \quad x_j \in \Sigma^*, \quad B_{j(k_j)} \in V,$$

$$0 \leqq j \leqq n \text{ for some } n \geqq 0\} \cup$$

$$\cup \{X \to X: X \in \Sigma \cup \{F\} \cup \{[a, 0]: a \in \Delta\} \cup \{[A, i]: A \in \Sigma \text{ and } 1 \leqq i \leqq k\}\} \cup$$

$$\cup \{[A, 0] \to F: A \in \Sigma \setminus \Delta\};$$

$$P_2 = \{[A, i] \to [A, i-1]: A \in \Sigma \text{ and } 1 \leqq i \leqq k\} \cup$$

$$\cup \{[A, 0] \to F: A \in \Sigma\} \cup \{X \to X: X \in \Sigma \cup \{F\}\};$$

$$P_3 = \{A \to \alpha: A \to \alpha \in P \text{ and } \alpha \in \Sigma^*\} \cup \{X \to X: X \in \Sigma \cup \{F\} \cup \bar{\Sigma}\}$$

and

$$P_4 = \{X \to F: X \in \Sigma \setminus \Delta \cup \bar{\Sigma} \setminus \bar{\Delta} \cup \{F\}\} \cup \{a \to a: a \in \Delta\} \cup$$

$$\cup \{[a, i] \to a: [a, i] \in \bar{\Delta} (a \in \Delta, 0 \leqq i \leqq k)\}.$$

Consider the ETOL system

$$\bar{G} = (\bar{\Sigma} \cup \Sigma \cup \{F\}, P_1, P_2, P_3, P_4, [S, 0], J, \Delta).$$

From the construction it is clear that $L(G) = L(\bar{G})$; hence the theorem holds. □

In the end we want to mention that it is not known whether or not the inclusion $\mathscr{L}(\mathrm{E}) \subseteqq \mathscr{L}(\mathrm{ETOL})$ is proper.

6. Subfamilies of $\mathscr{L}(\mathrm{E})$

In this section we prove a few results about some special cases of evaluated grammars which were defined in Section 3.

Theorem 2. $\mathscr{L}(\mathrm{EOL}) = \mathscr{L}(\mathrm{BE})$.

Proof. 1. $\mathscr{L}(\mathrm{EOL}) \subseteqq \mathscr{L}(\mathrm{BE})$: Let

$$G = (\Sigma, P, S, \Delta)$$

be an EOL system. Define a new alphabet $\bar{\Delta}=\{\bar{a}: a\in\Delta\}$ and a coding

$$h:\Sigma^*\to((\Sigma\setminus\Delta\times\{1\})\cup(\bar{\Delta}\times\{1\}))^* \text{ defined by}$$

(i) $h(X)=X_{(1)}$ for all $X\in\Sigma\setminus\Delta$;

(ii) $h(X)=\bar{X}_{(1)}$ for all $X\in\Delta$.

We define a new set of productions

$$\bar{P}=\{h(A)\to h(\alpha):\ A\to\alpha\in P\}\cup\{\bar{a}_{(1)}\to a_{(0)}:\ a\in\Delta(\bar{a}\in\bar{\Delta})\}.$$

Consider the BEG

$$\bar{G}=(\Sigma\cup\bar{\Delta},\bar{P},S,\Delta).$$

Clearly $L(G)=L(\bar{G})$ and thus $\mathscr{L}(\text{EOL})\subseteqq\mathscr{L}(\text{BV})$.

2. $\mathscr{L}(\text{EOL})\supseteqq\mathscr{L}(\text{BE})$: Let

$$G=(\Sigma,P,S,\Delta)$$

be a BEG and, clearly, we may assume without loss of generality that every nonterminal in G is useful i.e. that for every $A\in\Sigma\setminus\Delta$ there exists a word $\alpha\in(((\Sigma\setminus\Delta)\times\{1\})\cup(\Delta\times\{0\})\cup\Delta)^*$ such that $A\to\alpha\in P$. Define a new alphabet $\Sigma'=\{A': A\in\Sigma\}$ and a new "block" symbol F. We define the substitution

$$g:\ (\Delta\cup((\Sigma\setminus\Delta)\times\{1\})\cup(\Delta\times\{0\}))^*\to(\Delta\cup\Sigma')^*$$

by

(i) $g(a)=\{a',a\}$ for all $a\in\Delta$;

(ii) $g(A_{(1)})=A'$ for all $A_{(1)}\in(\Sigma\setminus\Delta)\times\{1\}$;

(iii) $g(a_{(0)})=a$ for all $a_{(0)}\in\Delta\times\{0\}$.

Let

$$P'=\{A'\to g(\alpha):\ A\to\alpha\in P\}\cup$$
$$\cup\{a'\to a',a'\to a,\ a\to F:\ a\in\Delta\}\cup$$
$$\cup\{F\to F\}.$$

Now, let

$$G'=(\Sigma'\cup\Delta\cup\{F\},P',S',\Delta)$$

be an EOL system, then, clearly, $L(G)=L(G')$ and thus $\mathscr{L}(\text{EOL})\supseteqq\mathscr{L}(\text{BE})$. Hence, we have $\mathscr{L}(\text{EOL})=\mathscr{L}(\text{BE})$ and the theorem holds. □

From this proof we obtain:

Corollary 1. For every $L\in\mathscr{L}(\text{BE})$ there exists a BEG $G=(\Sigma,P,S,\Delta)$ such that $L(G)=L$ and $P\subseteqq(\Sigma\setminus\Delta)\times((\Delta\times\{0\})\cup((\Sigma\setminus\Delta)\times\{1\}))^*$.

It is a straightforward to prove the following four lemmas.

Lemma 1. $\mathscr{L}(\text{BE})\subset\mathscr{L}(\text{E})$.

Proof. The inclusion $\mathscr{L}(\text{BE})\subseteqq\mathscr{L}(\text{E})$ is an immediate consequence of the definitions of BEG and EG. That the inclusion is strict follows from Example 2 in Section 4 and Theorem 2. □

Lemma 2. $\mathscr{L}(\mathsf{BE}) \not\subseteq \mathscr{L}(\mathsf{RGE})$.

Proof. From the construction in the proof of Theorem 1 follows that $\mathscr{L}(\mathsf{RGE}) \subseteq \mathscr{L}_{\mathsf{FIN}}(\mathsf{ETOL})$. On the other hand, $L(G_3) \in \mathscr{L}(\mathsf{BE}) / \mathscr{L}_{\mathsf{FIN}}(\mathsf{ETOL})$; see Example 3 in Sect. 4. Hence, the lemma holds. □

Lemma 3. $\mathscr{L}(\mathsf{RGE}) \not\subseteq \mathscr{L}(\mathsf{BE})$.

Proof. It is an immediate consequence of Example 2 in Sect. 4 and Theorem 2. □

Lemma 4. $\mathscr{L}(\mathsf{RGE}) \subset \mathscr{L}(\mathsf{E})$.

Proof. Clear. □

It is quite clear that $\mathscr{L}(\mathsf{RG}) \subset \mathscr{L}(\mathsf{BRGE})$; see the definition and Example 1 in Sect. 4. We now prove this result:

Lemma 5. $\mathscr{L}(\mathsf{BRGE}) \subseteq \mathscr{L}(\mathsf{PRL})$.

Proof. Let

$$G = (\Sigma, P, S, \Delta)$$

be n-regular BRGEG for some $n \geq 1$.

From the definition of BRGEG it follows that in any sentential form (except the last one) of a derivation of any word from $L(G)$, no symbol $a_{(0)}$, $a \in \Delta \times \{0\}$, is contained. Thus, we can construct the following n-parallel right linear grammar:

$$\bar{G} = (\Sigma \cup \bar{\Delta}, \bar{P}, S, \Delta)$$

where

$$\bar{\Delta} = \{\bar{a}\colon a \in \Delta\}$$

and

$$\bar{P} = \{S \to X_1 \dots X_n\colon S \to X_{1_{(1)}} \dots X_{n_{(1)}} \in P,\ X_{i_{(1)}} \in \Sigma \setminus (\Delta \cup \{S\}) \times \{1\},\ 1 \leq i \leq n,\ n > 0\} \cup$$
$$\cup \{A \to aB\colon A \to aB_{(1)} \in P,\ a \in \Delta,\ A \in \Sigma \setminus (\Delta \cup \{S\}),\ B_{(1)} \in ((\Sigma \setminus (\Delta \cup \{S\})) \times \{1\})\} \cup$$
$$\cup \{A \to a\colon A \to a_{(0)} \in P,\ A \in \Sigma \setminus (\Delta \cup \{S\}),\ a \in (\Delta \times \{0\})\} \cup$$
$$\cup \{A \to a,\ A \to \bar{a},\ \bar{a} \to \bar{a},\ \bar{a} \to a\colon A \to a \in P,\ A \in \Sigma \setminus (\Delta \cup \{S\}),\ a \in \Delta\}.$$

Clearly $L(G) = L(\bar{G})$. The lemma is proved. □

Lemma 6. $\mathscr{L}(\mathsf{CF})$ and $\mathscr{L}(\mathsf{BRGE})$ are incomparable but not disjoint.

Proof. The lemma is a direct consequence of Example 1 (see Sect. 4), Lemma 5 and a diagram from Sect. 6 in [1]. □

Lemma 7. $\mathscr{L}(\mathsf{CF}) \not\subseteq \mathscr{L}(\mathsf{RGE})$.

Proof. By proof of Lemma 2, $\mathscr{L}(\mathsf{RGE}) \subseteq \mathscr{L}_{\mathsf{FIN}}(\mathsf{ETOL})$. But it is well-known that $\mathscr{L}(\mathsf{CF}) \not\subseteq \mathscr{L}_{\mathsf{FIN}}(\mathsf{ETOL})$ (see, e.g., [2]). Thus the lemma holds. □

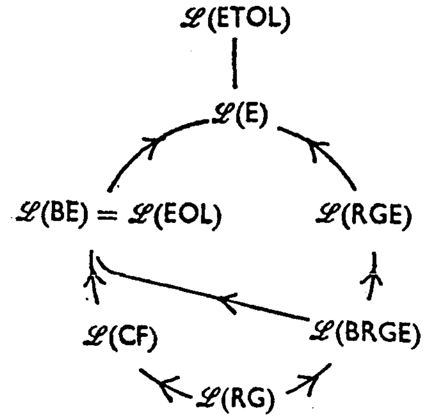
Lemma 8. $\mathscr{L}(\mathsf{BRGE}) \subseteq \mathscr{L}(\mathsf{BE}) \cap \mathscr{L}(\mathsf{RGE})$.

Proof. From definition. □

7. The relationship diagram

The aim of this section is to establish the relationship diagram among various classes of languages considered in this paper. We get the following theorem.

Theorem 3.



(If there is a directed chain of edges in the diagram leading from a class X to a class Y then $X \subset Y$, an undirected chain means that we do not know whether the inclusion is proper. Otherwise X and Y are incomparable but not disjoint.)

Proof. From the results of Sect. 5 and 6 together with the fact that $\mathscr{L}(\mathrm{RG}) \subset \subset \mathscr{L}(\mathrm{CF}) \subset \mathscr{L}(\mathrm{EOL})$ — see [2]. □

Abstract

Evaluated grammars are based on context free grammars but the derivation process in these grammars is regulated by a certain evaluation of some symbols occurring in their sentential forms.

Fundamental properties of the family of languages generated by evaluated grammars are investigated. This family of languages is contained in the family of ETOL languages and properly contains the family of EOL languages.

In addition, we propose and study some special cases of evaluated grammars.

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