Analysis of the SCAN service polling model

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Abstract

A performance analysis is given for a polling system in which the server polls N stations back and forth according to the so-called SCAN system. Messages arrive at each station in Poisson fashion at an average rate λ. The number of characters in a message has a geometric distribution with mean $1/\sigma$. The service time per character, b, and the switchover time between adjacent stations, r, are both assumed to be constant.

An exact analysis is given but because of associated computational problems this has limited application. Thus a second approximate analysis which allows systems with a large number of stations to be treated, has been developed. In both cases it is a simple matter to calculate such performance measures as average polling cycle time and mean response time at each station.

Keywords: Machine interference models, Markov chain, Performance evaluation, Polling models, SCAN system.

1 Introduction

We consider a SCAN polling model in which the server polls N stations in the order $1, 2, 3, \ldots, N-1, N, N-1, N-2, \ldots, 3, 2, 1, 2, 3$, etc. Messages arrive at each station in Poisson fashion at an average rate λ and the number of characters in a message has a geometric distribution with mean $1/\sigma$. Thus at the completion of each character service the message completes its service with probability σ, and does not complete service with probability $1-\sigma$. Yet another viewpoint is that we have single buffer stations, and are modelling error-prone transmission channels. Here σ is the probability that the message departs (successful transmission). The service time per character, b, and the switchover time between adjacent stations, r, are both assumed to be constants.

Polling systems have been the subject of much study for a number of years. A good review and bibliography is given by Takagi [8,10] who also mentions the SCAN system as well as the geometric distribution of characters referred to as a round-robin system in Takagi [9]. For an up-to-date bibliography see Takagi [11]. Coffman and Gilbert [4] consider a continouos polling model of SCAN service. Other analyses of this system can be found in the work of Coffman and Hofri [5], Swartz [7] and Takagi and Murata [12]. In fact the SCAN system has also been considered in the context of machine interference models by Bunday and Mack [3] and Bunday, El-Badri and Supanekar [1]. Similar mathematical models have also

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been treated by Kim and Koenigsberg [6] in their analysis of automatic loading and retrieval on carousel conveyor systems.

2 Exact Analysis

The assumption of Poisson arrival of messages at average rate λ means that if a station is empty at time t, then the chance that it is still empty after a further period of time T is

$$\exp(-\lambda T) \tag{1}$$

whereas the chance that a message has arrived by this time is

$$1 - \exp(\lambda T) \tag{2}$$

independent of T.

We denote by

$$\{u_1/u_2u_3\dots u_N\}$$

the left to right traverse of the stations (i.e. in the order $1, 2, \dots, N-1, N$) in which the server *leaves* station 1 in state u_1 and *finds* station i ($i = 2, \dots, N$) in state u_i, where $u_i = 0$ denotes no message at station i, whereas $u_i = 1$ indicates that there is a message at station i.

$$p\{u_1/u_2u_3\dots u_N\}$$

is the probability that the server encounters this situation on a left to right traverse.

Similarly right to left traverses of the stations, when they are polled in the order $N, N-1, \dots, 3, 2, 1$, occur with probability (in an obvious notation)

$$p\{v_1v_2\dots v_{N-1}/v_N\}$$

and because of the symmetry of the system,

$$p\{v_1v_2\dots v_{N-1}/v_N\} = p\{v_N/v_{N-1}\dots v_2v_1\}. \tag{3}$$

The 2^N probabilities $p\{u_1/u_2\dots u_N\}$ satisfy the condition

$$\sum_{\{u_1/u_2\dots u_N\}} p\{u_1/u_2\dots u_N\} = 1, \tag{4}$$

along with the usual equilibrium equation for the finite Markov chain, viz.

$$p\{u_1/u_2\dots u_N\} = \sum_{\{v_1v_2\dots v_{N-1}/v_N\}} p\{v_1v_2\dots v_{N-1}/v_N\}P\{u_1/u_2\dots u_N|v_1v_2\dots v_{N-1}/v_N\}, \tag{5}$$

which on account of (3) can be written

$$p\{u_1/u_2\dots u_N\} = \sum_{\{v_1/v_2\dots v_N\}} p\{v_1/v_2\dots v_N\} P\{u_1/u_2\dots u_N | v_N v_{N-1}\dots v_2/v_1\}, \tag{6}$$

for each $\{u_1/u_2\dots u_N\}$.

Here $P\{u_1/u_2\dots u_N|v_N v_{N-1}\dots v_2/v_1\}$ is the conditional probability of encountering the situation described by $\{u_1/u_2\dots u_N\}$ on the left to right traverse following the right to left traverse described by $\{v_N v_{N-1}\dots v_2/v_1\}$.

The computation of this transition probability proceeds along the following lines. For the jth station, the time that elapses between it being left on a right to left traverse and next visited on the following left to right traverse, is just $2r + bv_N$ for $j = 2$, and

$$\tau_j = 2(j-1)r + b\left[\sum_{i=2}^{j-1} u_i + \sum_{i=N-j+2}^{N} v_i\right] \tag{7}$$

for $j = 3, 4, \dots, N$.

If we denote by $Pr_j(u_j|v_{N-j+1})$ the probability that the server finds station j in state u_j on a left to right traverse, having found it in state v_{N-j+1} on the previous right to left traverse, then from (1) and (2), for $j = 2, 3, \dots N-1$,

$$Pr_j(0|0) = \exp(-\lambda\tau_j), \qquad Pr_j(1|0) = 1 - \exp(-\lambda\tau_j)$$

$$Pr_j(0|1) = \sigma\exp(-\lambda\tau_j), \qquad Pr_j(1|1) = 1 - \sigma\exp(-\lambda\tau_j). \tag{8}$$

For $j = 1$ we need the probability of *leaving* station 1 in state u_1 having just *found* it in state v_N. Thus

$$Pr_1(0|0) = 1 \quad Pr_1(1|0) = 0, \quad Pr_1(0|1) = \sigma, \quad Pr_1(1|1) = 1 - \sigma \tag{9}$$

For $j = N$ we consider the probability of *finding* station N in state u_N having *left* it in state v_1. Thus

$$Pr_N(0|0) = \exp(-\lambda\tau_N), \qquad Pr_N(1|0) = 1 - \exp(\lambda\tau_N)$$

$$Pr_N(0|1) = 0, \qquad Pr_N(1|1) = 1. \tag{10}$$

There is some abuse of notation here since $Pr_j(u_j|v_{N-j+1})$ also depends on the other $u's$ and $v's$ through τ_j. We use (7), (8), (9) and (10) to compute the transition probability as

$$P\{u_1/u_2\dots u_N|v_N\dots v_2/v_1\} = \Pi_{j=1}^{N} Pr_j(u_j|v_{N-j+1}). \tag{11}$$

We use (4) and (6) to obtain 2^N independent equations for the 2^N state probabilities. This analysis can be used in conjunction with modern computing facilities to obtain exact solutions for $N \leq 10$.

It is perhaps worth mentioning that provided we have symmetry in the "line of stations" so that (3) holds then the same analysis will hold for the inhomogeneous system with message arrival rate λ_j and character service time b_j at station j. We simply replace λ by the appropriate λ_j in (8) and (10) whilst (7) is replaced by

$$r_j = 2(j-1)r + \sum_{i=2}^{j-1} b_i u_i + \sum_{i=N-j+2}^{N} b_{N-i+1} v_i.$$

For further details and applications to machine interference models the reader is referred to the report by Bunday and Khorram (2).

3 Approximate Analysis

The modelling in the previous section calls for the solution of 2^N simultaneous linear equations. In the present state of computing this limits the method to values of $N \leq 10$.

A traverse of the stations begins when the server leaves polling station $1(N)$ and ends when he leaves station $N(1)$. Thus at most $N-1$ characters are serviced on a traverse. A complete cycle consists of two consecutive traverses.

In the equilibrium state the probability that station m is message free when polled from the left is denoted by

$$p_m \quad (2 \leq m \leq N) \tag{12}$$

The probability that station m is message free when polled from the right is denoted by p'_m and from the symmetry

$$p'_m = p_{N+1-m} \quad (m = N-1, N-2, \ldots, 2, 1). \tag{13}$$

$$p_m = \Sigma^{(m)} p\{u_1/u_2 \ldots u_N\} \tag{14}$$

where $\Sigma^{(m)}$ denotes summation over those states for which $u_m = 0$.

For $m \geq 2$ the left partial cycle time (L.P.C.T.) of station m is defined to be the time that elapses between the server's departure from station m and subsequent arrival at this station, the journey being made via station 1. It does not include the service time, if any, for station m, on either occasion. The right partial cycle time (R.P.C.T) in which the journey is made via station N is similarly defined. In the equilibrium state the L.P.C.T. of station m and the R.P.C.T. of station $N+1-m$ have identical distributions.

We let

$$u(l, m) \quad (2 \leq m \leq N, 0 \leq l \leq 2m-3) \tag{15}$$

denote the probability that in the L.P.C.T. of station m, l characters are serviced (l of the stations have messages). Thus $u(l, m)$ is the equilibrium probability that the L.P.C.T. of station m takes the value

$$q(l, m) = 2(m-1)r + bl. \tag{16}$$

Similarly

$$v(l, m) \tag{17}$$

is the probability that the L.P.C.T. of station m takes the value $q(l, m)$ given that station m had no message, when it was approached from the right just prior to the left partial cycle, whereas

$$w(l,m) \tag{18}$$

is the probability that the L.P.C.T. of station m takes the value $q(l,m)$ given that station m had a message just before the partial cycle.

Thus

$$u(l,m) = p'_m v(l,m) + (1 - p'_m) w(l,m)$$

i.e.

$$u(l,m) = p_{N+1-m} v(l,m) + (1 - p_{N+1-m}) w(l,m) \tag{19}$$

on using (13).

For station 2, $u(0,2) = p'_1$ and $u(1,2) = 1 - p'_1$ so that

$$u(0,2) = p_N \text{ and } u(1,2) = 1 - p_N. \tag{20}$$

On using (1) we obtain

$$\begin{aligned} p_2 \;=\; & p'_2 v(0,2) \exp\left[-\lambda q(0,2)\right] + (1 - p'_2)\sigma w(0,2) \exp\left[-\lambda q(0,2)\right] \\ & + p'_2 v(1,2) \exp\left[-\lambda q(1,2)\right] + (1 - p'_2)\sigma w(1,2) \exp\left[-\lambda q(1,2)\right], \end{aligned}$$

i.e.,

$$\begin{aligned} p_2 \;=\; & p_{N-1}\{v(0,2)\exp[-\lambda q(0,2)] + v(1,2)\exp[-\lambda q(1,2)]\} \\ & + (1 - p_{N-1})\sigma\{w(0,2)\exp - [\lambda q(0,2)] + w(1,2)\exp[-\lambda q(1,2)]\} \end{aligned} \tag{21}$$

For $2 < m \leq N$ it is possible to relate the distribution of the L.P.C.T. for station m to that of station $(m-1)$. On using (1), (2) and (13) we obtain for $2 < m \leq N$

$$\begin{aligned} u(0,m) \;=\; & p_{N+2-m} v(0, m-1) \exp[-\lambda q(0, m-1)] \\ u(1,m) \;=\; & p_{N+2-m} v(1, m-1) \exp[-\lambda q(1, m-1)] \\ & + p_{N+2-m} v(0, m-1)\{1 - \exp[-\lambda q(0, m-1)]\} \\ & + (1 - p_{N+2-m})\sigma w(0, m-1) \exp[-\lambda q(0, m-1)] \end{aligned}$$

and for $1 < l < 2m - 4$

$$\begin{aligned} u(l,m) \;=\; & p_{N+2-m} v(l, m-1) \exp[-\lambda q(l, m-1)] \\ & + p_{N+2-m} v(l-1, m-1)\{1 - \exp[-\lambda q(l-1, m-1)]\} \\ & + (1 - p_{N+2-m})\sigma w(l-1, m-1) \exp[-\lambda q(l-1, m-1)] \\ & + p_{N+2-m} v(l-2, m-1)\{1 - \exp[-\lambda q(l-2, m-1)]\} \\ & + (1 - p_{N+2-m})(1-\sigma) w(l-2, m-1) \end{aligned} \tag{22}$$

$$\begin{aligned} u(2m-4, m) \;=\; & p_{N+2-m} v(2m-5, m-1)\{1 - \exp[-\lambda q(2m-5, m-1)]\} \\ & + (1 - p_{N+2-m})\sigma w(2m-5, m-1) \exp[-\lambda q(2m-5, m-1)] \\ & + (1 - p_{N+2-m})\sigma w(2m-6, m-1) \\ & \times\{1 - \exp[-\lambda q(2m-6, m-1)]\} \\ & + (1 - p_{N+2-m})(1-\sigma) w(2m-6, m-1) \\ u(2m-3, m) \;=\; & (1 - p_{N+2-m})\sigma w(2m-5, m-1) \\ & \times\{1 - \exp[-\lambda q(2m-5, m-1)]\} \\ & + (1 - p_{N+2-m})(1-\sigma) w(2m-5, m-1) \end{aligned}$$

For station m, (21) generalises to

$$\begin{aligned} p_m &= p_{N+1-m} \sum_{l=0}^{2m-3} v(l,m) \exp[-\lambda q(l,m)] \\ &\quad + (1-p_{N+1-m})\sigma \sum_{l=0}^{2m-3} w(l,m) \exp[-\lambda q(l,m)]. \end{aligned} \tag{23}$$

Now for each m the probabilities $u(l,m)$ form a complete distribution so that

$$\sum_{l=0}^{2m-3} u(l,m) = 1. \tag{24}$$

For $m = 2$ the result is clearly true from (20) and using (19) and (22) and some elementary but messy algebra the general result is easily established by induction.

The exact relationship between $v(l,m)$ and $w(l,m)$ is enormously complicated and involves the probabilities $p\{u_1/u_2 \ldots u_N\}$. To bring these latter quantities into consideration makes the problem impossible from a computational aspect if N is large. However if N is large and/or λb small the conditioning effect of the state of one station will become insignificant. In this situation (19) shows that

$$u(l,m) = v(l,m) = w(l,m). \tag{25}$$

Thus for $2 < m \leq N$, (22) takes the form,

$$\begin{aligned} u(0,m) &= p_{N+2-m} u(0,m-1) \exp[-\lambda q(0,m-1)] \\ u(1,m) &= p_{N+2-m} u(1,m-1) \exp[-\lambda q(1,m-1)] \\ &\quad + p_{N+2-m} u(0,m-1)\{1-\exp[-\lambda q(0,m-1)]\} \\ &\quad + (1-p_{N+2-m})\sigma u(0,m-1) \exp[-\lambda q(0,m-1)] \end{aligned}$$

and for $1 < l < 2m-4$,

$$\begin{aligned} u(l,m) &= p_{N+2-m} u(l,m-1) \exp[-\lambda q(l,m-1)] \\ &\quad + p_{N+2-m} u(l-1,m-1)\{1-\exp[-\lambda q(l-1,m-1)]\} \\ &\quad + (1-p_{N+2-m})\sigma u(l-1,m-1) \exp[-\lambda q(l-1,m-1)] \\ &\quad + (1-p_{N+2-m}) u(l-2,m-1)\{1-\sigma \exp[-\lambda q(l-2,m-1)]\} \end{aligned} \tag{26}$$

$$\begin{aligned} u(2m-4,m) &= p_{N+2-m} u(2m-5,m-1)\{1-\exp[-\lambda q(2m-5,m-1)]\} \\ &\quad + (1-p_{N+2-m})\sigma u(2m-5,m-1) \exp[-\lambda q(2m-5,m-1)] \\ &\quad + (1-p_{N+2-m}) u(2m-6,m-1) \\ &\quad \times\{1-\sigma \exp[-\lambda q(2m-6,m-1)]\} \\ u(2m-3,m) &= (1-p_{N+2-m}) u(2m-5,m-1) \\ &\quad \times\{1-\sigma \exp[-\lambda q(2m-5,m-1)]\}. \end{aligned}$$

The *extended* version of (23) becomes

$$p_1 = p_N + \sigma(1-p_N)$$

$$p_m = (p_{N+1-m} + \sigma - \sigma p_{N+1-m}) \sum_{l=0}^{2m-3} u(l,m) \exp[-\lambda q(l,m)], \quad 1 < m < N, \quad (27)$$

$$p_N = p_1 \sum_{l=0}^{2m-3} u(l,N) \exp[-\lambda q(l,N)].$$

Here p_1 has been defined as the probability that station 1 is *left* without a message on a left to right traverse.

It is possible to solve (20), (26) and (27) numerically. Just $N^2 - 1$ unknown quantities are involved. From an initial approximation for $p_1, p_2, \ldots, p_N$ (derived from the solution for the previous lower value of N) it is possible to use the equations recursively to compute a better approximation. In this way the p_m were calculated to an accuracy of 10^{-6}.

Of course for values of $N \leq 10$ the exact and approximate analysis can be compared. In neither case are the required computations trivial, but as expected the calculations indicated that the approximate analysis is exact for $N = 2$, is at its worst for N around 6, 7 and 8, but improves again at $N = 9$ and 10. One might add that in all situations the errors involved were quite small, the more so for the performance measures considered in the next section. The relative values of λb and λr had an effect on the validity of the approximate analysis.

Indeed when $b = 0$ the approximate analysis is again exact. Any delays in this case are due to switchover times in order to reach stations with messages and to leaving stations with messages following unsuccessful transmission. In this case (27) takes the form

$$p_1 = p_N + \sigma(1 - p_N)$$

$$p_m = c^{m-1}(p_{N-m+1} + \sigma - \sigma p_{N-m+1}), \quad 2 \leq m \leq N-1, \quad (28)$$

$$p_N = c^{N-1} p_1$$

where $c = \exp(-2\lambda r)$. These equations are easily solved in pairs. It is perhaps worth commenting that this particular solution is not immediately apparent from the exact analysis of Section 2.

4 Performance Measures

The mean time τ for the server to traverse the stations in one direction is given by

$$\tau = (N-1)r + b \sum_{\{u_1/u_2 \ldots u_N\}} p\{u_1/u_2 \ldots u_N\} S\{u_1/u_2 \ldots u_N\} \quad (29)$$

where

$$S\{u_1/u_2 \ldots u_N\} = \sum_{i=2}^{N} u_i \quad (30)$$

is the number of character services carried out on the traverse $\{u_1/u_2 \ldots u_N\}$. The mean time for a complete cycle is just 2τ.

If we use (14) a second more useful form for τ is

$$\tau = (N-1)r + b\sum_{j=2}^{N}(1-p_j). \tag{31}$$

The mean time that station m is free of messages in its L.P.C.T. is

$$\begin{aligned} L_m \;=\; & p_{N+1-m}\sum_{l=0}^{2m-3} v(l,m)\left\{\int_0^{q(l,m)} \lambda t e^{-\lambda t}dt + q(l,m)\exp[-\lambda q(l,m)]\right\} \\ & +\sigma(1-p_{N+1-m})\sum_{l=0}^{2m-3} w(l,m)\left\{\int_0^{q(l,m)} \lambda t e^{-\lambda t}dt + q(l,m)\exp[-\lambda q(l,m)]\right\} \end{aligned}$$

Thus, on using (23), for $2 \le m \le N-1$ we have

$$L_m = \frac{\sigma + (1-\sigma)p_{N+1-m} - p_m}{\lambda}. \tag{32}$$

If F_m denotes the mean time that station m is free of messages in a complete cycle

$$F_m = L_m + L_{N+1-m} = \frac{\sigma[1-p_m+1-p_{N+1-m}]}{\lambda}. \tag{33}$$

for $2 \le m \le N-1$
while

$$F_1 = F_N = \frac{\sigma(1-p_N)}{\lambda}. \tag{34}$$

Thus if β_m is the proportion of time that station m is free of messages (not blocked)

$$\beta_m = F_m/2\tau. \tag{35}$$

The major performance measure at station m is the mean message response time, which is the average time that an arbitrary message, arriving at the station, takes from its arrival to its service completion. We denote this by $\mathrm{E}[T_m]$. Other performance measures such as the mean waiting time $\mathrm{E}[W_m]$ and throughput γ_m are easily calculated from $\mathrm{E}[T_m]$

$$\mathrm{E}[T_m] = \mathrm{E}[W_m] + b, \tag{36}$$

$$\gamma_m = 1\Big/\left[\mathrm{E}[T_m] + \frac{1}{\lambda}\right]. \tag{37}$$

But $\mathrm{E}[T_m]$ can be calculated from the result

$$\beta_m = \frac{1}{\lambda}\Big/\left[\frac{1}{\lambda} + \mathrm{E}[T_m]\right]$$

whence

$$\lambda \mathrm{E}[T_m] = \frac{1}{\beta_m} - 1. \tag{38}$$

The case of zero switchover times gives a system identical, insofar as the mean response time is concerned, to the M/D/1/N queue with first-come, first served queue discipline. In that case

$$\lambda \mathrm{E}[T] = \lambda N b - 1 + 1 \Big/ \left[1 + \sum_{n=1}^{N-1} \begin{bmatrix} N-1 \\ n \end{bmatrix} \prod_{j=1}^{n} (e^{\lambda j b} - 1) \right]. \tag{39}$$

5 Some Numerical Results

The problem has five parameters N, σ, λ, b and r and in consequence coverage of a wide range of values would make extensive demands on space in any tables. Suffice it to say that the computer program which did the calculations easily deals with other values of the above parameters.

Table 1 gives details of the performance measures at individual stations and illustrates the inhomogeneity of the performance along the line.

In Table 1 the column headed $L\beta_m$ records the quantity $L_m/2\tau$. From (35) β_m represents the proportion of time that station m is not blocked and since of course

$$\beta_m = L\beta_m + L\beta_{N+1-m} \tag{40}$$

for $2 \le m \le N-1$, $L\beta_m$ indicates how this proportion is divided between the left partial cycles and the right partial cycles.

Table 2 details the system performance. The proportion of time on average that stations in the system are not blocked is

$$\beta = \frac{\sum_{m=1}^{N} \beta_m}{N} \tag{41}$$

The average time that a message at a station takes from its arrival to service completion is given by

$$\lambda \mathrm{E}[T] = \frac{1}{\beta} - 1. \tag{42}$$

Table 1

Performance Measures at Individual Stations

$\sigma = 0.1$

N	λb	$N\lambda r$	m	$L\beta_m$	β_m	$\lambda E[T_m]$	p_m
4	0.01	0.01	2	0.2738	0.8251	0.2120	0.9110
			3	0.5513	0.8251	0.2120	0.9078
			4	0.7583	0.7583	0.3187	0.8335
7	0.01	0.01	2	0.1247	0.7703	0.2982	0.8672
			3	0.2550	0.7704	0.2980	0.8647
			4	0.3852	0.7704	0.2980	0.8623
			5	0.5154	0.7704	0.2980	0.8598
			6	0.6456	0.7703	0.2982	0.8574
			7	0.6770	0.6770	0.4771	0.7579
12	0.01	0.01	2	0.0534	0.6755	0.4804	0.7273
			3	0.1167	0.6757	0.4799	0.7244
			4	0.1799	0.6759	0.4796	0.7215
			5	0.2432	0.6760	0.4794	0.7186
			6	0.3064	0.6760	0.4792	0.7158
			7	0.3696	0.6760	0.4792	0.7130
			8	0.4328	0.6760	0.4794	0.7102
			9	0.4959	0.6759	0.4796	0.7074
			10	0.5590	0.6757	0.4799	0.7047
			11	0.6221	0.6755	0.4804	0.7020
			12	0.5361	0.5361	0.8655	0.5471
18	0.01	0.01	2	0.0229	0.5157	0.9392	0.5474
			3	0.0544	0.5159	0.9383	0.5441
			4	0.0858	0.5161	0.9375	0.5409
			5	0.1172	0.5163	0.9369	0.5377
			6	0.1486	0.5164	0.9364	0.5345
			7	0.1800	0.5165	0.9360	0.5314
			8	0.2113	0.5166	0.9357	0.5282
			9	0.2427	0.5166	0.9356	0.5252
			10	0.2740	0.5166	0.9356	0.5221
			11	0.3053	0.5166	0.9357	0.5191
			12	0.3365	0.5165	0.9360	0.5162
			13	0.3678	0.5164	0.9364	0.5132
			14	0.3990	0.5163	0.9369	0.5103
			15	0.4303	0.5161	0.9375	0.5074
			16	0.4615	0.5159	0.9383	0.5046
			17	0.4927	0.5157	0.9392	0.5018
			18	0.3576	0.3576	1.7963	0.3406

Table 1

Performance Measures at Individual Stations

$\sigma = 0.1$

N	λb	$N\lambda r$	m	$L\beta_m$	β_m	$\lambda \mathrm{E}[T_m]$	p_m
4	0.05	0.05	2	0.1272	0.4039	1.4756	0.5281
			3	0.2768	0.4039	1.4756	0.5094
			4	0.2813	0.2813	2.5552	0.3298
7	0.05	0.05	2	0.0353	0.2690	2.7176	0.3230
			3	0.0853	0.2696	2.7090	0.3076
			4	0.1349	0.2698	2.7061	0.2935
			5	0.1843	0.2696	2.7090	0.2804
			6	0.2337	0.2690	2.7176	0.2683
			7	0.1628	0.1628	5.1419	0.1473
12	0.05	0.05	2	0.0103	0.1634	5.1212	0.1882
			3	0.0270	0.1640	5.0985	0.1760
			4	0.0433	0.1644	5.0817	0.1649
			5	0.0591	0.1647	5.0706	0.1547
			6	0.0747	0.1649	5.0650	0.1454
			7	0.0902	0.1649	5.0650	0.1369
			8	0.1056	0.1647	5.0706	0.1292
			9	0.1212	0.1644	5.0817	0.1222
			10	0.1370	0.1640	5.0985	0.1158
			11	0.1531	0.1634	5.1212	0.1099
			12	0.0906	0.0906	10.0342	0.0559
18	0.05	0.05	2	0.0046	0.1094	8.1420	0.1368
			3	0.0124	0.1099	8.1033	0.1264
			4	0.0197	0.1102	8.0709	0.1169
			5	0.0267	0.1106	8.0444	0.1082
			6	0.0334	0.1108	8.0235	0.1003
			7	0.0400	0.1110	8.0080	0.0931
			8	0.0463	0.1111	7.9978	0.0866
			9	0.0525	0.1112	7.9927	0.0807
			10	0.0587	0.1112	7.9927	0.0753
			11	0.0648	0.1111	7.9978	0.0704
			12	0.0711	0.1110	8.0080	0.0660
			13	0.0774	0.1108	8.0235	0.0620
			14	0.0838	0.1106	8.0444	0.0583
			15	0.0905	0.1102	8.0709	0.0550
			16	0.0975	0.1099	8.1033	0.0520
			17	0.1048	0.1094	8.1420	0.0493
			18	0.0588	0.0588	15.9995	0.0245

Table 2

System Performance Measures

N	λb	$N\lambda r$	β	$\lambda E[T]$
4	0.01	0.01	0.7917	0.2631
7	0.01	0.01	0.7437	0.3446
12	0.01	0.01	0.6525	0.5325
18	0.01	0.01	0.4986	1.0054
4	0.05	0.05	0.3426	1.9187
7	0.05	0.05	0.2390	3.1849
12	0.05	0.05	0.1520	5.5789
18	0.05	0.05	0.1048	8.5435

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(Received February 3, 1990)