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Symbiotic E0L Systems

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Summary: Cell symbiosis is described by E0L systems whose (direct) derivations are introduced on free monoids generated by finite sets of words consisting of one or two symbols. A single symbol represents a cell existing separately while two cells living symbiotically are represented by a pair of symbols. By using these systems, context sensitive and recursively enumerable languages are characterized. Thus, the presented modification remarkably increases the generative capacity of the classic concept of E0L systems.

Key words: Cell symbiosis - E0L systems - Monoids - Generative capacity

1 Introduction

1.1 Three points of view

From the *biological point of view,* this paper attempts to describe cell symbiosis (see [1]-[2]) in a simple and formal way. To do so, the classic concept of E0L systems (see [3]-[4], [11]) is modified so that their derivations are introduced on free monoids generated by finite sets of words consisting of one or two symbols. Single symbols represent cells living separately while pairs of symbols represent cells living symbiotically. Attempting to propose our formal model as universal as possible, we do not differentiate between associations of plant-plant, animal-animal, and plant-animal cells or between prokaryotic and eukaryotic cells. It is proved that our approach remarkably increases the generative capacity of E0L systems. Or, more biologically speaking, it allows us to describe some developments of cell organisms than the classical approach does not.

From the *mathematical point of view,* such a generalization of E0L systems is very natural: rather that allowing only letter monoids as domains of derivations we now introduce the derivations on free monoids generated by words consisting of one or two symbols. In other words, we investigate single finite substitutions iterated on free monoids generated by finite number of words having one or two symbols.

From the *formal language theory point of view,* the resulting grammars are very simple in comparison with some other rewriting mechanisms (see, eg, [5]-[7], [9] and references therein). Moreover, they characterize both context sensitive and recursively enumerable languages in a natural way.

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1.2 Relation to some other rewriting systems

Although there are some similarities between our generalization of E0L systems and EIL systems (see [11]), both of the concepts are fundamentally different: in EIL systems the way a letter is rewritten depends on its neighbors while in our systems it does not. What is restricted in our approach are derivation domains.

There are also some analogies between this paper and [8]. The latter introduces the notion of a derivation on word monoids generated by finite sets of words over total vocabularies of context free grammars. By using generators of length at most two, context sensitive and recursively enumerable languages are characterized by such modification of context-free grammars. The analogical result is proved for the generalisation of E0L systems in this paper. Thus, we get the same generative power of both parallel and sequential context-independent rewriting defined on free monoids generated by finite words having one or two symbols. Since both ways of rewriting generate quite different language families when defined on letter monoids (see [11]), this result may be of some interest.

2 Preliminaries

2.1 Basic Notions

We assume that the reader is familiar with formal language theory (see [12]), in particular, with the theory of L systems (see [11]). Some notations and definitions need perhaps an additional explanation.

For a vocabulary V, V^* denotes the letter monoid (generated by V under the operation of concatenation), e is the unit of $V^*, V^+ = V^* - \{e\}$. For a word $x \in V^*, |x|$ denotes the length of x. For a finite set of words W over V, W^* denotes the word monoid (generated by W under the operation of concatenation).

A context free grammar is a quadruple $G = (V, P, S, T)$, where, as usual, V is a finite alphabet, P is a finite set of productions of the form $A \longrightarrow x$, where $A \in V - T, x \in V^*, S \in V - T$ is the axiom, and $T \subseteq V$ is a terminal alphabet.

A context sensitive grammar is specified in Penttonen Normal Form $G = (V, P, S, T)$, where V, S, and T have the same meaning as for a context free grammar and every production in P is either of the form $AB \longrightarrow AC$ or $A \longrightarrow x$, where $A, B, C \in V - T, x \in (T \cup (V - T)^2)$, see Theorem 2 in [10].

A phrase structure grammar is also specified in Penttonen Normal Form $G = (V, P, S, T)$, where V, S, and T are as for a context free grammar and every production in P is either of the form $AB \longrightarrow AC$ or $A \longrightarrow x$, where $A, B, C \in V - T, x \in (\{e\} \cup T \cup (V - T)^2)$, see Theorem 4 in [10].

Given a (context free, context sensitive, or phrase structure) grammar G, in the standard manner we can introduce the relations $\Longrightarrow, \Longrightarrow^i, \Longrightarrow^+$, and $\Longrightarrow^*$ on the free monoid generated by its alphabet. If we want to express that $x \Longrightarrow y$ in G according to production p, then we write $x \Longrightarrow y\ [p]$.

2.2 Basic Definition

We now introduce a new concept of E0L systems, the subject of investigation in this paper, namely the notion of a symbiotic E0L system.

Let V be an alphabet. A *symbiotic E0L system* (SE0L-system for short) is a 4-tuple $G = (W, P, S, T)$, where $W \subseteq (V \cup V^2)$, P is a finite set of productions of the form $A \longrightarrow x$, where $A \in V, x \in V^*, S \in V - T$, and $T \subseteq V$. G is said to be *propagating* if $A \longrightarrow x \in P$ implies $x \neq e$. G is called an *E0L system* if $V = W$.

The direct derivation relation $\Longrightarrow$ is now defined on W^* as follows: For arbitrary words $x, y \in W^*$ such that $x = a_1a_2 \ldots a_n, a_i \in V, y = y_1y_2 \ldots y_n, y_i \in V^*$, and productions $a_1 \longrightarrow y_1, a_2 \longrightarrow y_2, \ldots, a_n \longrightarrow y_n \in P$, we say that x *directly derives* y *according to* $a_1 \longrightarrow y_1, a_2 \longrightarrow y_2, \ldots, a_n \longrightarrow y_n$ (in G), in symbols

$$x \Longrightarrow yx \Longrightarrow y \quad [a_1 \longrightarrow y_1, a_2 \longrightarrow y_2, \ldots, a_n \longrightarrow y_n]$$

The list of applied productions (written in the brackets) is usually omitted when no confusion arises. We denote the i-fold product of $\Longrightarrow$ (for some $i \geq 0$) by $\Longrightarrow^i$, the transitive closure of $\Longrightarrow$ by $\Longrightarrow^+$, and the reflexive and transitive closure of $\Longrightarrow$ by $\Longrightarrow^*$. The *language of* G, denoted by $L(G, W)$, is defined by $L(G) = \{v \in T^* : S \Longrightarrow^* v\}$.

2.3 Denotation of Language Families

We denote by **SE0L** and **E0L** the families of languages generated by SE0L- and E0L-systems, respectively. The families of languages generated by propagating SE0L- and E0L-systems are denoted by **SEP0L** and **EP0L**, respectively. The family of context-free, context-sensitive, and recursively enumerable languages are denoted by **CF**, **CS**, and **RE**, respectively.

3 Results

3.1 Aim and Preliminary Results

In this section, we examine the generative capacity of (propagating) SE0L-systems. It follows immediately from the definitions and some basic results of formal language theory (see [11]-[12]) that

$$\mathbf{CF} \subset \mathbf{EP0L} = \mathbf{E0L} \subset \mathbf{CS} \subset \mathbf{RE}$$

and

$$\mathbf{EP0L} = \mathbf{E0L} \subseteq \mathbf{SEP0L} \subseteq \mathbf{SE0L}$$

Next we will give precision to these relationships when proving that

(i) a language is context sensitive if and only if it is generated by a propagating SE0L-system and

(ii) a language is recursively enumerable if and only if it is generated by a SE0L-system.

We close Section 3.1 by recalling a technical lemma from [8] yielding a normal form for context-sensitive grammars similar to the one given by Penttonen (see Section 2.1). We will find it useful when proving Theorem 1.

Lemma 1 *Every* $L \in \mathbf{CS}$ *can be generated by a context sensitive grammar* $G = (N_{CF} \cup N_{CS} \cup T, P, S, T)$, *where* N_{CF}, N_{CS}, *and* T *are pairwise disjoint alphabets and every production in* P *is either of the form* $AB \longrightarrow AC$, *where* $B \in N_{CS}, A, C \in N_{CF}$, *or of the form* $A \longrightarrow x$, *where* $A \in N_{CF}, x \in N_{CS} \cup T \cup N_{CF}^2$

3.2 Main Result

Now we consider the relationship between **CS** and **SEP0L**.

Theorem 1 **CS = SEP0L.**

Proof. It is straightforward to prove that **SEP0L** $\subseteq$ **CS**, hence it suffices to prove the converse inclusion.

Let L be a context sensitive language generated by a context sensitive grammar $G = (N_{CF} \cup N_{CS} \cup T, P, S, T)$ of the form described by Lemma 1.

Let
$$V = (N_{CF} \cup N_{CS} \cup T)$$
and
$$V' = V \cup Q,$$
where
$$Q = \{<A, B, C>: AB \longrightarrow AC \in P, A, C \in N_{CF}, B \in N_{CS}\}.$$
Clearly, without loss of generality, we can assume that $Q \cap V = \emptyset$.

The SEP0L-grammar, G', is defined as follows:
$$G' = (W, P', S, T),$$
where the set of productions P' is defined in the following way:

(0) for all $A \in V'$, add $A \longrightarrow A$ to P';
(1) if $A \longrightarrow x \in P, A \in N_{CF}, x \in N_{CS} \cup T \cup N_{CF}^2$, then add $A \longrightarrow x$ to P';
(2) if $AB \longrightarrow AC \in P, A, C \in N_{CF}, B \in N_{CS}$, then add the set of two productions $\{B \longrightarrow <A, B, C>, <A, B, C> \longrightarrow C\}$ to P'.

The set $W \subseteq (V \cup V^2)$ is defined as follows:
$$W = \{A <A, B, C>: <A, B, C> \in Q(A \in N_{CF})\} \cup V.$$
Obviously G is an SEP0L grammar.

Let us now introduce a function h from $(V')^*$ into V^* defined by:
for all $D \in V$, $h(D) = D$,
for all $<X, D, Z> \in Q$, $h(<X, D, Z>) = D$;
let h^{-1} be the inverse of h.

To show that $L(G) = L(G')$, we first prove two claims:

Claim 1 *If $S \Longrightarrow^m w$ in $G, w \in V^+$, for some $m \geq 0$, then $S \Longrightarrow^* v$ in G', where $v \in h^{-1}(w)$.*

Proof of Claim 1: This is established by induction on the length m of derivations in G.

Let $m = 0$. The only w is S because $S \Longrightarrow^0 S$ in G. Since $S \in W^*, S \Longrightarrow^0 S$ in G' and by the definition of $h^{-1}, S \in h^{-1}(S)$.

Let us suppose that our claim holds for all derivations of length at most m for some $m \geq 0$ and consider a derivation $S \Longrightarrow^{m+1} x$ in $G, x \in V^*$. Since $m + 1 \geq 1$, there is some $y \in V^+$ and $p \in P$ such that $S \Longrightarrow^m y \Longrightarrow x[p]$ in G and by the induction hypothesis there is also a derivation $S \Longrightarrow^n y'$ in G' for some $y' \in h^{-1}(y), n \geq 0$. By the definition, $y' \in W^*$.

(i) Let us first assume that $p = D \longrightarrow y_2 \in P, D \in N_{CF}, y_2 \in N_{CS} \cup T \cup N_{CF}^2, y = y_1 D y_3$, and $x = y_1 y_2 y_3, y_1 = a_1 \dots a_i, y_3 = b_1 \dots b_j$, where $a_k, b_l \in V, 1 \leq k \leq i, 1 \leq l \leq j$, for some $i, j \geq 0$ ($i = 0$ implies $y_1 = e$ and $j = 0$ implies $y_3 = e$). Since from the definition of h^{-1} it is clear that $h^{-1}(Z) = \{Z\}$ for all $Z \in N_{CF}$ we can write $y' = z_1 D z_3$, where $z_1 \in h^{-1}(y_1)$ and $z_3 \in h^{-1}(y_3)$, that is to say, $z_1 = c_1 \dots c_i, z_3 = d_1 \dots d_j$, where $c_k \in h^{-1}(a_k)$, $d_l \in h^{-1}(b_l)$, for $1 \leq k \leq i, 1 \leq l \leq j$. It is clear that $D \longrightarrow y_2 \in P'$, see (1).

Let $d_1 \notin Q$. Then, it is easy to see that $z_1 y_2 z_3 \in W^*$ and so

$$z_1 D z_3 \Longrightarrow z_1 y_2 z_3 \; [c_1 \longrightarrow c_1, \dots, c_i \longrightarrow c_i, D \longrightarrow y_2, d_1 \longrightarrow d_1, \dots, d_j \longrightarrow d_j]$$

in G'. Therefore, $S \Longrightarrow^n z_1 D z_3 \Longrightarrow z_1 y_2 z_3$ and $z_1 y_2 z_3 \in h^{-1}(y_1 y_2 y_3)$.

Let $d_1 \in Q$, that is, $Dh(d_1) \longrightarrow DC \in P$ (for some $C \in N_{CF}$), see the definition of h. Hence, we have $h(d_1) \longrightarrow d_1$ in P', see (2) (observe that this production is the only production in P' that has d_1 appearing on its right-hand side). It is clear, by the definition of W, that $d_2 \notin Q$. Thus, $\{z_1 D h(d_1) d_2 \dots d_j, z_1 y_2 h(d_1) d_2 \dots d_j\} \subseteq W^*$. Since $S \Longrightarrow^n z_1 D d_1 \dots d_j$ in G', there must exist the following derivation in G':

$$\begin{array}{lll} S \Longrightarrow^{n-1} & z_1 D h(d_1) d_2 \dots d_j & \\ \Longrightarrow & z_1 D d_1 d_2 \dots d_j & [c_1 \longrightarrow c_1, \dots, c_i \longrightarrow c_i, D \longrightarrow D, \\ & & h(d_1) \longrightarrow d_1, d_2 \longrightarrow d_2, \dots, d_j \longrightarrow d_j] \end{array}$$

in G'. So, we get

$$\begin{array}{lll} S \Longrightarrow^{n-1} & z_1 D h(d_1) d_2 \dots d_j & \\ \Longrightarrow & z_1 y_2 h(d_1) d_2 \dots d_j & [c_1 \longrightarrow c_1, \dots, c_i \longrightarrow c_i, D \longrightarrow y_2, \\ & & h(d_1) \longrightarrow h(d_1), d_2 \longrightarrow d_2, \dots, d_j \longrightarrow d_j] \end{array}$$

such that $z_1 y_2 h(d_1) d_2 \dots d_j$ is in $h^{-1}(x)$.

(ii) Let $p = AB \longrightarrow AC \in P, A, C \in N_{CF}, B \in N_{CS}, y = y_1 A B y_2, y_1, y_2 \in V^*, x = y_1 A C y_2, y' = z_1 A Y z_2, z_i \in h^{-1}(y_i), i \in 1, 2, Y \in h^{-1}(B)$, and $y_1 = a_1 \dots a_i, y_3 = b_1 \dots b_j, a_k, b_l \in V, 1 \leq k \leq i, 1 \leq l \leq j$, for some $i, j \geq 0$. Let $z_1 = c_1 \dots c_i, z_3 = d_1 \dots d_j, c_k \in h^{-1}(a_k), d_l \in h^{-1}(b_l), 1 \leq k \leq i, 1 \leq l \leq j$. Clearly, $\{B \longrightarrow < A, B, C >, < A, B, C > \longrightarrow C\} \subseteq P'$, see (2), and $A < A, B, C > \in W$, see the definition of W.

Let $Y = B$. Since $y' \in W^*$ and $B \in N_{CS}, d_1 \notin Q$. Consequently, $z_1 A < A, B, C > z_2$ and $z_1 A C z_2$ are in W^* by the definition of W. Thus,

$$\begin{array}{lll} S \Longrightarrow^n & z_1 A B z_2 & \\ \Longrightarrow & z_1 A < A, B, C > z_2 & [c_1 \longrightarrow c_1, \dots, c_i \longrightarrow c_i, A \longrightarrow A, \\ & & B \longrightarrow < A, B, C >, d_1 \longrightarrow d_1, \dots, d_j \longrightarrow d_j] \end{array}$$

$$\begin{array}{ll} \Longrightarrow z_1 A C z_2 & [c_1 \longrightarrow c_1, \dots, c_i \longrightarrow c_i, A \longrightarrow A, \\ & < A, B, C > \longrightarrow C, d_1 \longrightarrow d_1, \dots, d_j \longrightarrow d_j] \end{array}$$

and $z_1 A C z_2 \in h^{-1}(x)$.

Let $Y \in Q$. Clearly, $h(Y)$ must be equal to B. By (2) and the definition of Q, we have $B \longrightarrow Y \in P'$. Clearly, $z_1 A C z_2$ is in W^* for $d_1 \notin Q$ as we have already shown. Thus, since $S \Longrightarrow^n z_1 A Y z_2$ in G', the word $z_1 A Y z_2$ can be derived in G' as follows:

$$\begin{array}{lll} S \Longrightarrow^{n-1} & z_1ABz_2 & \\ \Longrightarrow & z_1AYz_2 & [c_1 \longrightarrow c_1, \dots, c_i \longrightarrow c_j, A \longrightarrow A, \\ & & B \longrightarrow Y, d_1 \longrightarrow d_1, \dots, d_j \longrightarrow d_j]. \end{array}$$

Since $z_1A < A, B, C > z_2$ and z_1ACz_2 belong to W^*, we get

$$\begin{array}{lll} S \Longrightarrow^{n-1} & z_1ABz_2 & \\ \Longrightarrow & z_1A < A, B, C > z_2 & [c_1 \longrightarrow c_1, \dots, c_i \longrightarrow c_i, A \longrightarrow A, \\ & & B \longrightarrow < A, B, C >, d_1 \longrightarrow d_1, \dots, d_j \longrightarrow d_j] \end{array}$$

$$\begin{array}{ll} \Longrightarrow z_1ACz_2 & [c_1 \longrightarrow c_1, \dots, c_i \longrightarrow c_i, A \longrightarrow A, \\ & < A, B, C > \longrightarrow C, d_1 \longrightarrow d_1, \dots, d_j \longrightarrow d_j] \end{array}$$

in G', where $z_1ACz_2 \in h^{-1}(x)$.

The points (i) and (ii) cover all possible rewriting of y in G. Thus, the claim now follows by the principle of induction.

Claim 2 *If $S \Longrightarrow^n u$ in $G', u \in W^*$, for some $m \geq 0$, then $S \Longrightarrow^* h(u)$ in G.*

Proof of Claim 2: This is established by induction on the length m of derivations.

For $n = 0$ the only u is S because $S \Longrightarrow^0 S$ in G'. Since $S = h(S)$ we have $S \Longrightarrow^0 S$ in G.

Let us assume the claim holds for all derivations of length at most n, for some $n \geq 0$, and consider a derivation $S \Longrightarrow^{n+1} u$, where $u \in W^*$. Since $n+1 \geq 1$, there is some $v \in W^*$, such that $S \Longrightarrow^n v \Longrightarrow u[p]$ in G' and by the induction hypothesis $S \Longrightarrow^* h(v)$ in G.

We will first prove the following statement (*):

Let $v = rDs$ and $p = D \longrightarrow z \in P$ in G'. Then $h(v) \Longrightarrow^i h(r)h(z)h(s)$ in G, for some $i = 0, 1$. (*)

To verify (*), consider the following three cases:

(i) Let $h(z) = h(D)$, see (2). Then $h(v) \Longrightarrow^0 h(r)h(z)h(s)$ in G.

(ii) Let $z \in (T \cup N_{CS} \cup N_{CF}^2), D \in N_{CF}$ Then there is a production $B \longrightarrow z \in P$, see (1), and by the definition of h we have $B \longrightarrow z = h(B) \longrightarrow h(z)$. Thus, $h(r)h(D)h(s) \Longrightarrow h(r)h(z)h(s)[h(B) \longrightarrow h(z)]$ in G.

(iii) Let $z = C \in N_{CF}$ and $D =< A, B, C >$ for some $< A, B, C > \in Q$, see (2). By the definition of W, we have $r = tA$, where $t \in W^*$ and so $v = tACs$. By the definition of Q, there is a production $AB \longrightarrow AC \in P$. Thus, $tABs \Longrightarrow tACs[AB \longrightarrow AC]$ in G where $tABs = h(tA)h(< A, B, C >)h(s)$ and $tACs = h(tA)h(C)h(s)$.

By inspection of P', the points (i) through (iii) cover all possible types of productions in P', proving (*).

It should be clear that by using (i) through (iii) we can construct the derivation $h(v) \Longrightarrow^i h(u)$, for some $i \in \{0, \dots, |u|\}$, in the following way: first we rewrite all occurrences of symbols corresponding to the case (iii) and then all occurrences of symbols corresponding to (ii); the technical details are left to the reader.

Thus, $S \Longrightarrow^n h(v) \Longrightarrow^i h(u)$ in G. Hence, by the principle of induction, we have established Claim 2.

Now, the proof of the equivalence of G and G' can be derived from Claim 1 and 2:

By the definition of h^{-1}, we have $h^{-1}(a) = \{a\}$ for all $a \in T$. Thus, by Claim 1, we have for any $x \in T^*$:

$$S \Longrightarrow^* x \text{ in } G \text{ implies } S \Longrightarrow^* x \text{ in } G',$$

that is, $L(G) \subseteq L(G')$.

Conversely, since $T^* \subseteq W^*$, we get, by the definition of h and Claim 2, for any $x \in T^*$:

$$S \Longrightarrow^* x \text{ in } G' \text{ implies } S \Longrightarrow^* x \text{ in } G,$$

that is, $L(G') \subseteq L(G)$.

Thus, $L(G) = L(G')$ and so **CS** = **SEP0L**, which proves the theorem.

□

3.3 Some Corollaries an Conclusion

First of all, Theorem 1 and the definitions yield the following normal form:

Corollary 1 *Let L be a context sensitive language over an alphabet T. Then L can be generated by an SEP0L system $G = (W, T, P, S)$, where W is over an alphabet V such that $T \subseteq W, (W-V) \subseteq (V-T)^2$, and if $A \longrightarrow x$ and $1 < |x|$ then $x \in (V-T)^2$.*

Let us now turn to the investigation of SE0L-grammars with erasing productions. We will show that these grammars generate precisely the family of recursively enumerable languages.

Corollary 2 **RE** = **SE0L**.

Proof. Clearly we have the containment **SE0L** $\subseteq$ **RE**, hence it suffices to show **RE** $\subseteq$ **SE0L**.

Each language $L \in$ **RE** can be generated by a phrase structure grammar G in Penttonen Normal Form (see Section 2.1) which can be converted to the grammar of an analogical form to the one described by Lemma 1 (except that the former may contain some erasing productions), see [8]. Thus, the containment **RE** $\subseteq$ **SE0L** can be proved by analogy with the techniques used in the proof of Theorem 1. The details are left to the reader.

□

Since the forms of the resulting SE0L-grammar in the proof of Corollary 2 and that in the proof of Theorem 1 are analogical, we get the following:

Corollary 3 *Let L be a recursively-enumerable language over an alphabet T. Then L can be generated by an SE0L system $G = (W, T, P, S)$, where W is over an alphabet V such that $T \subseteq W, (W-V) \subseteq (V-T)^2$, and if $A \longrightarrow x$ and $1 < |x|$ then $x \in (V-T)^2$.*

Finally, summing up the main results of this paper, we obtain:

Corollary 4 **SEP0L** = **CS** $\subset$ **SE0L** = **RE**.

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