Mathematical models for simulation of continuous grinding process with recirculation

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Abstract

New mathematical and computer models and simulation programs were elaborated for studying processes of continuous grinding mills working with classification and partial recirculation of the product. The computer models were developed on the basis of the axial dispersion model taking into consideration also the effects of the mixing of the material to be ground. The effects of changes of parameters of both the mill and the material were studied. The stationary states of the continuous grinding mills working with and without classification and recirculation were compared to each other. The mathematical models and the computer programs developed are suitable for computing the processes of the grinding mills either with or without recirculation. They are usable for simulation based analysis and design of both continuous and batch grinding devices.

1 Introduction

Grinding is an important technological process in process industry. The two basic types of grinding are the batch and the continuous grinding. The mathematical analysis of batch grinding has been carried out in a number of aspects [1-9]. Considerable experimental research has been taken on continuous grinding [10-13], too, but fewer results were published concerning the mathematical modelling of the continuous grinding processes.

The mathematical description of continuous grinding mills can be formulated by means of distributed parameter models. One of these types of models, derived from the integro-differential equation of continuous grinding, was published and verified by Mihálykó *et al.* [13]. By using this discrete type model, the effects of system parameters on the behaviour and performance of the mills can be studied extensively. However, the classification and partial recirculation of the material to be ground, that is an important and often used solution for increasing the quality of the product of griders, was not included into this model.

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The aim of the present paper is to develop a generalised model, taking into consideration the classification and recirculation processes of the material to be ground. The kinetics of grinding is modelled by the fundamental grinding equation. The computer models presented are based on the resulting partial integro-differential equation. The newly developed computer models are suitable for simulation of continuous grinding mills working with and without classification and recirculation.

2 Mathematical model of continuous grinding with recirculation

In order to review the theory let us introduce the following notation. Let t denote time, t_r - the time spent by material in the recycling loop, X - length of the mill, x - axial coordinate of the mill, L - particle size, $\nu(x,t)$ - the average linear velocity of the particles in the mill, $D(x,t)$ - the axial dispersion coefficient characterising the mixing of particles in the mill, and $m(x,L,t)$ - the mass density function of particles in the mill. The mass density function characterises the size distribution of particles by means of which $m(x,L,t)\mathrm{d}L$ expresses the mass of particles at axial coordinate x of the mill at time of t within the particle size interval (L, $L+\mathrm{d}L$) in a unit volume of the mill. Let $m_{in}(L,t)$ denote the mass density function of the particles entering the mixing device, $m_r(L,t_r)$ the mass density function of the particles leaving the classifier and entering the mixing device again.

Let $f(L,t)$ denote the mass flow density function of the particles leaving the mill, as it shown in Fig.A, $f_f(L,t)$ denotes the mass flow density function of the particles entering the mill, and $f_{out}(L,t)$ is the mass flow density function of the particles leaving the grinding system. Let $f_r(L,t)$ denote the mass flow density function of the particles classified and recycled with the delay time t_r. Furthermore, let $f_{in}(L,t)$ be the mass flow density function of the particles fed into the system, and $\psi(L,t)$ be the selection function describing the classifying device. All of these flows are presented in Fig.A, illustrating schematically the structure of the whole continuous grinding system with classification and recirculation.

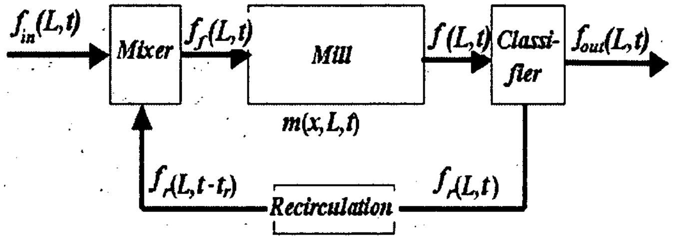


Fig.A. Continuous grinding system with classification and recirculation

Using the above notation, the mathematical model of the grinding mill can be written as a partial integro-differential equation:

$$\begin{aligned}\frac{\partial m(x,L,t)}{\partial t} &= \frac{\partial}{\partial x}\left(D(x,t)\frac{\partial m(x,L,t)}{\partial x}\right) - \frac{\partial v(x,t)m(x,L,t)}{\partial x} - \\ &\quad -S(L,t)m(x,L,t) + \int_{L}^{L_{max}} S(l,t)b(L,l)m(x,l,t)dl\end{aligned} \tag{1}$$

subject to the following initial and boundary conditions:

$$m(x,L,0) = m_0(x,L) \tag{2}$$

$$f_f(L,t) = v(x,t)m(x,L,t) - D(x,t)\frac{\partial m(x,L,t)}{\partial x}, \quad if\ x = 0 \tag{3}$$

$$D(x,t)\frac{\partial m(x,L,t)}{\partial x} = 0, \quad if\ x = X \tag{4}$$

The left-hand side of Eq.(1) describes the rate of accumulation, the first term on the right-hand side represents the axial dispersion, while the second term corresponds to the convective flow of particles in the axial direction of the mill. The third and fourth terms of the right-hand side of Eq.(1) describe the rates of changes of particles due to the grinding process. The selection function $S(L,t)$ represents the rate of breakage of particles of size L. By means of function $b(l,L)$, called breakage density function, $b(l,L)\mathrm{d}L$ expresses the mass fraction of the product of size $(L, L+\mathrm{d}L)$ when particles of size l are broken. Based on that, $B(l;\lambda)$ denotes the breakage distribution function which expresses the mass fraction of particles of size l broken into the size interval $[L_{min}, \lambda)$, where L_{min} is the grindability limit of the mill. As a consequence, $B(l,\lambda) = \int_{L_{\min}}^{\lambda} b(l,L)dL$.

The initial condition (2) expresses the fact that the mill is assumed to be filled with solids characterised with mass density function m_0 at the beginning of the process. Boundary conditions (3) and (4) reflect the continuity of particle fluxes at the inlet and that of the mass density function at the outlet of the mill, respectively. In boundary condition (3), the left-hand size term represents the flow from the transfer pipe into the mill, which is equal to the flow density of particles, consisted of convective and dispersive parts described by the right-hand side of Eq.(3). This boundary condition expresses the assumption that there is no back-mixing from the mill into the transfer pipe. At the exit boundary X, the particles are assumed to flow from a mixed region to a region where there is no mixing at all, so that the composition suffers no change here, and the boundary condition at the outlet reduces to the form of Eq.(4).

A continuous grinding system with classification and recirculation can be operated in various ways. Here we consider two models. In both cases, the material to be ground is fed into the mill through a mixer in which the mass flow $f_{in}(L,t)$ of the fresh raw material, as well as the mass flow $f_r(L, t-t_r)$ of large size fractions recycled from the classifier, are mixed. In the first case, however, called model number I, the total mass flow of the fresh raw material is constant and does not depend on the amount of the recycled particulate product. Since the mass flow of the recycled material depends on the operating parameters of the mill and classifier, this mode, in general, leads to time dependent loading of the mill. This type of grinding system with classified product removal can be described by the following equations:

$$\begin{aligned} f(L,t) &= v(X,t)m(X,L,t) && (5)\\ f_r(L,t) &= \psi(L,t)v(X,t)m(X,L,t) && (6)\\ f_{out}(L,t) &= [1-\psi(L,t)]\,v(X,t)m(X,L,t) && (7)\\ f_{in}(L,t)+f_r(L,t-t_r) &= f_f(L,t) && (8) \end{aligned}$$

Eq.(5) describes the mass flow density at the outlet of the mill. Eq.(6) expresses, by means of the selection function $\psi(L,t)$ characterising in principle the operation of the classifying device, that fraction of the particulate product that consists of particles having sizes larger than required and, as a consequence, are returned into the mill again. The quantity of returned particles therefore is determined by the selection function. Eq.(7) describes that part of the particulate product that is fine enough to leave the grinding process. Finally, Eq.(8) describes the mixture of particles resulted in mixing of the fresh raw material fed into the system and of the recycled mass flow by the mixing device. Also, Eq.(8) expresses that there is a time delay t_r in the recycled stream caused by the classifier device and the transport of particles through the recirculation line.

The second operation mode of continuous grinding systems with selective recirculation, considered here and called model number II, is as follows. The total mass flow of the material fed freshly into the grinding system is controlled in time according to the actual mass flow of the recycled particles, that may be variable in time depending on the operating conditions of the mill and the classifying device, in order to have constant loading of the mill itself. This type of operation of the continuous grinding system with selective recirculation is also described by Eq.(1) subject to the initial and boundary conditions (2)-(8), but it satisfy also the following constraint:

$$\int_{L_{\min}}^{L_{\max}} f_f(l,t)dl = constant. \qquad (9)$$

where L_{max} denotes the maximal size of the particles to be ground by the mill.

The main difference between the two models is that model number I describes such a case when the mean residence time of the particles in the mill is varied depending on the actual size distribution of the fresh raw material fed into the system and on the operation conditions of the mill and classifying device. Conversely, in the second case the mean residence time of the material to be ground in the mill is constant by applying some appropriate control of the flow of raw material fed into the grinding system.

3 Discrete mathematical model of continuous grinding with classification and recirculation

In order to develop the computer models suitable for simulation of continuous grinding process with classification and recirculation, we had to discretise the partial integro-differential equation (1), describing the transport and breakage of the material to be ground in the mill itself, subject to the initial and boundary conditions (2)-(8). The discretization method used here is based on that developed and presented by Mihálykó *et al.* [13]. This type of discrete models has proved very useful in modelling and computer simulation of both batch and continuous grinding processes without classification.

Let us introduce the following notation. Let T denote the final time of a grinding process. We subdivide the interval $[0,T]$ into N equal subintervals, let τ denote the length of a subinterval, and let $t_n = n \cdot \tau$. We subdivide also the interval $[L_{min}, L_{max}]$ representing the extent of sizes of the whole particle population, into I equal subintervals, and let l_i denote the ith size fraction of those. The interval $[0,X]$ represents the length of the mill, and we subdivide it into J equal subintervals. Let h denote the length of a subinterval, and let $x_j = j \cdot h$. After the discretization process, we consider the mill as consists of imaginary columns. Let $V(x_j, l_i, t_n)$ denote the quantity of the particles belong to the ith particle size fraction in the jth column of the mill at t_n moment of time t. Further, let V_F denote the velocity of the material to be ground forward, while V_B the velocity of that backward in axial direction, respectively, so that $(V_F - V_B)$ is the velocity of the convective flow in the axial direction in the mill. Let a_i denote the quantity of the freshly fed particles of size l_i, r_i the rate of the returned part of the mass for for particles of size l_i leaving the mill, where $0 \le r_i \le 1$, and d the discretised time delay in the recirculation line.

As concerns the kinetics of breakage, usually the discretised versions of functions S and B, defined in Eq.(1) for the continuous case, are used for that purpose. Namely, here we chosen $S(l_k) = K_S \cdot (l_k)^n$ and $B(l_k, l_i) = (l_i / l_k)^m$, where $S(l_k)$ denotes the breakage selection function, representing the specific rate of breakage of particles of size l_k with parameters n and K_S, and $B(l_k, l_i)$ with parameter m denotes the breakage distribution function interpreted as fraction of breakage product from size interval l_k which falls into size interval l_i.

The discrete model number I of the continuous grinding system with classification and partial recirculation of product is as follows.

The quantities of particles in the first column of the mill at the moment of time t_{n+1} are expressed as

$$\begin{aligned} V(x_1, l_i, t_{n+1}) &= (1 - V_F) \cdot (1 - S(l_i)) \cdot V(x_1, l_i, t_n) + V_B \cdot V(x_2, l_i, t_n) \\ &+ (1 - V_F) \cdot \sum_{k=i}^{I} p_{k,i} \cdot V(x_1, l_k, t_n) + a_i \qquad (10) \\ &+ r_i \cdot V(x_J, l_i, t_{n-d}) \qquad i = 1, 2, \ldots, I \end{aligned}$$

where the first term on the right-hand side, $(1-V_F)\cdot(1-S(l_i))\cdot V(x_1,l_i,t_n)$, gives the amount of those particles from the ith size subinterval at the moment of time t_n which have not moved forward and have not broken. The second term, $V_B\cdot V(x_2,l_i,t_n)$ represents the quantity of those particles from the ith size subinterval at the moment of time t_n which have moved backward from the second column. The third term, $(1-V_F)\cdot\sum_{k=i}^{I} p_{k,i}\cdot V(x_1,l_k,t_n)$, expresses the quantity of those particles from the ith size subinterval at the same moment of time which have not moved forward from the first column to the second one, but have broken from some size subinterval to the ith subinterval of particle size. The term a_i is the quantity of the freshly fed particles belonging to the ith subinterval. Finally, the last term, $r_i\cdot V(x_J,l_i,t_{n-d})$, gives the quantity of the particles fed again into the mill because of their recycling.

The quantities of particles in some inner column of the mill can be described at the moment of time t_{n+1} as

$$\begin{aligned} V(x_j,l_i,t_{n+1}) &= (1-V_F-V_B)\cdot(1-S(l_i))\cdot V(x_j,l_i,t_n) \\ &\quad + V_F\cdot V(x_{j-1},l_i,t_n) + V_B\cdot V(x_{j+1},l_i,t_n) \qquad (11) \\ &\quad +(1-V_F-V_B)\cdot\sum_{k=i}^{I} p_{k,i}\cdot V(x_j,l_k,t_n) \\ &\qquad j=2,\dots,J-1,\ i=1,2,\dots,I \end{aligned}$$

The first term on the right-hand side of Eq.(11), $(1-V_F-V_B)\cdot(1-S(l_i))\cdot V(x_j,l_i,t_n)$, expresses the quantity of those particles from the ith subinterval at the moment of time t_n which have moved neither forward nor backward and have not broken. The second term, $V_F\cdot V(x_{j-1},l_i,t_n)$, represents the quantity of particles from the ith size subinterval at the moment of time t_n which have moved forward from the previous column. The third term, $V_B\cdot V(x_{j+1},l_i,t_n)$, gives the quantity of particles from the ith size subinterval at the moment of time t_n which have moved backward from the next column. Finally, the last term, $(1-V_F-V_B)\cdot\sum_{k=i}^{I} p_{k,i}\cdot V(x_j,l_k,t_n)$, represents the quantity of those particles which have moved neither forward nor backward from the jth column, but have broken from some size subinterval to the ith one larger from that.

At the last, the discretised process in the last column of the mill at the moment of time t_{n+1} can be given as

$$\begin{aligned} V(x_J,l_i,t_{n+1}) &= (1-V_F)\cdot(1-S(l_i))\cdot V(x_J,l_i,t_n) \\ &\quad + V_F\cdot V(x_{J-1},l_i,t_n) \qquad (12) \\ &\quad + (1-V_F)\cdot\sum_{k=i}^{I} p_{k,i}\cdot V(x_J,l_k,t_n) \qquad i=1,2,\dots,I \end{aligned}$$

The first term on the right-hand side of Eq.(12) expresses the quantity of those particles which have not moved forward from the last column, and they have not broken, i.e. there is no back mixing from the last column. The second term represents the amount of those particles which have moved forward from the last column, i.e. these particles have left the mill at the outlet at the moment of time t_n. The third term gives the quantity of those particles, which have not moved forward from the last column, but have broken from some particle size subinterval to the ith one. In Eqs (10)-(12) the meaning of symbols $p_{k,i}$ is, in principle,

$$p_{k,i} = \tau \cdot S(l_k) \cdot [B(l_k, l_i) - B(l_k, l_{i-1})] \text{ for all } k \text{ and } i.$$

The discrete model number II, considered in this paper, differs from that given by Eqs (10)-(12) only in the equation describing the processes occurring in the first column of the discretised version of the mill. Namely, the quantities of particles in the first column of the mill at the moment of time t_{n+1} are expressed as

$$\begin{aligned} V(x_1, l_i, t_{n+1}) &= (1 - V_F) \cdot (1 - S(l_i)) \cdot V(x_1, l_i, t_n) + V_B \cdot V(x_2, l_i, t_n) \\ &+ (1 - V_F) \cdot \sum_{k=i}^{I} p_{k,i} \cdot V(x_1, l_k, t_n) + a_i \qquad\qquad (10/a) \\ &+ r_i \cdot V(x_J, l_i, t_{n-d}) \qquad\qquad i = 1, 2, \ldots, I \end{aligned}$$

where now $a_i = a_i(t_n)$ $i = 1, 2, \ldots, I$ and

$$\sum_{i=1}^{I} (a_i(t_n) + r_i \cdot V(x_J, l_i, t_{n-d})) \text{ is constant.} \qquad (13)$$

Eq.(13) is the discretised version of Eq.(9). Since the second term on right hand side of Eq.(11), $V_F \cdot V(x_{j-1}, l_i, t_n)$, represents the influence of a column to the next one, the changes in the first column move smoothly to the other ones of the mill.

4 Simulation results and discussion

Based on the discretised equations (10)-(13), two computer programs, written in the language C, were developed for numerical experimentation. The size distributions of both the fed material and the initial loading material were chosen to be monodisperse in the simulation runs. The sizes of particles fed into the grinding system were chosen larger than $L_{max}/2$. At the same time, classification of the product was assumed to be total and sharp at particle size $L_{max}/2$, i.e. the selection function was chosen the Heaviside function of the form $\psi(L, t) = 1 \cdot (L - L_{max}/2)$ so that all particles larger than $L_{max}/2$ were returned from the classifier to the mixer.

The cumulative size distribution of the material being in the mill at coordinate x_j of the mill and that of the product leaving the mill were computed for all moments of time t_n ($n = 0, 1, 2, \ldots N$) as the sum

$$M_j(L,t_n) = \sum_{l_i \le L} V(x_j, l_i, t_n) \quad (j = 1, 2, \ldots, J-1), \quad \text{and}$$
$$M_J(L,t_n) = \sum_{l_i \le L} V(x_J, l_i, t_n),$$

respectively. The oversize distribution functions were obtained by means of the relations $R_j(L,t_n) = 1 - M_j(L,t_n)$ ($j = 1, 2, \ldots J$-1) and $R_J(L,t_n) = 1 - M_J(L,t_n)$.

Eq.(1) describing the continuous grinding process may become independent of the time when $t \to \infty$. In this case, the stationary state is achieved when the dependence on time becomes negligible. Then, the size distribution and the total mass of the material leaving the mill also become independent on time. There may exist, however, such operation conditions of the grinding system, mostly due to the size dependent removal of the product, that the system does not achieve stationary state. In this case, the amount of the material to be ground in the mill increasing monotonically what leads to overloading of the mill after elapsing some time. Overloading is a heavy breakdown, and the mill must be stopped.

The simulation program was used to examine how the classification and partial recirculation, as well as the time delay in the recirculation line affect the stationary state and the oversize distribution of the product leaving the mill. The effects of changes of parameters both of the mill and the material to be ground were also examined. Let us see a few examples.

The effects of the classification and recirculation, and of the variation of the delay parameter d on the duration of transients and on some characteristic parameters of the size distribution of the particulate product is presented in Tables 1 and 2 for two different values of parameter K_s of the breakage selection function.

It is well seen from these tables that the duration of time required for reaching the stationary state is increased considerably with increasing the time delay, whilst the average size and dispersion of the size distribution are reduced. Table 1 contains simulation results for material the size reduction of which is easier, $K_s = 10^{-7}$, than that of the material with parameter $K_s = 0.5 \cdot 10^{-7}$ the results of which are shown in Table 2. Comparing data in Tables 1 and 2 allows concluding that the extent of reduction of both the average size and the dispersion is smaller in the case of easy-to-grind material than in the opposite case. The extent of time delay influences the time required for reaching the stationary state, the average size and dispersion of the size distribution of the product significantly. The results obtained for cases $d=3$ and $d=10$ are very similar.

Using the same process and kinetic parameters as before, we obtained results shown in Tables 3 and 4 for the discrete model number II. It is seen that the effects of the recycling are not as much significant as in the case of the discrete model number I. In principle, in the case of the discrete model number II, the duration of time of transients is increased only to negligible extent, while only small changes can be observed in the average size and the dispersion of the size distribution of the material leaving the mill.

Table 1: Dependence of the duration of time required for reaching the stationary state, and of the characteristic parameters of the size distribution of the material to be ground at the outlet of the mill at the stationary state. Discrete model number I. Parameters: I=20, J=30, m=3, n=2, V_F =0.5, V_B=0.1, L_{max}=2000.

Recirculation, Delay	Ks	Time required for reaching the stationary state	The average size of the material leaving the mill	The dispersion of the material leaving the mill
No	10^{-7}	86	644.91	270.22
Yes, d=3	10^{-7}	145	635.15	267.45
Yes, d=5	10^{-7}	145	635.23	267.52
Yes, d=10	10^{-7}	145	635.38	267.68

Table 2: Dependence of the duration of time required for reaching the stationary state, and of the characteristic parameters of the size distribution of the material to be ground at the outlet of the mill at the stationary state. Discrete model number I. Parameters: I=20, J=30, m=3, n=2, V_F =0.5, V_B=0.1, L_{max}=2000.

Recirculation, Delay	Ks	Time required for reaching the stationary state	The average size of the material leaving the mill	The dispersion of the material leaving the mill
No	$0.5 \cdot 10^{-7}$	82	855.15	334.47
Yes, d=3	$0.5 \cdot 10^{-7}$	155	830.54	321.10
Yes, d=5	$0.5 \cdot 10^{-7}$	158	830.61	321.15
Yes, d=10	$0.5 \cdot 10^{-7}$	166	830.68	321.25

Table 3: Dependence of the duration of time required for reaching the stationary state, and of the characteristic parameters of the size distribution of the material to be ground at the outlet of the mill at the stationary state. Discrete model number II. Parameters: I=20, J=30, m=3, n=2, V_F =0.5, V_B=0.1, L_{max}=2000.

Recirculation, Delay	Ks	Time required for reaching the stationary state	The average size of the material leaving the mill	The dispersion of the material leaving the mill
No	10^{-7}	86	644.91	270.22
Yes, d=3	10^{-7}	87	639.54	265.43
Yes, d=5	10^{-7}	87	639.61	265.46
Yes, d=10	10^{-7}	88	639.38	265.59

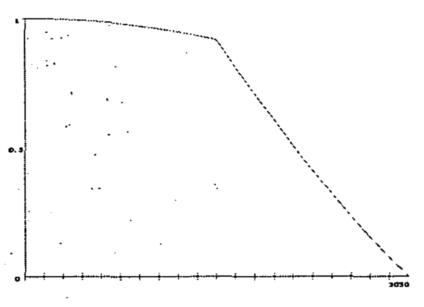
As a second example, we examine the oversize distribution of the leaving material as a function of time by using the discrete model number I. We compare the oversize distribution functions of the leaving material obtained by simulating the

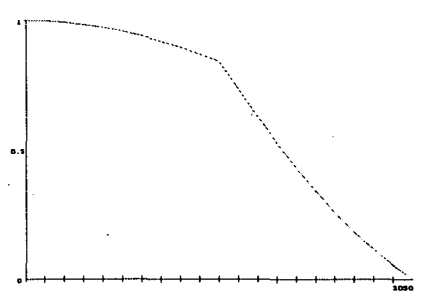
Table 4: Dependence of the duration of time required for reaching the stationary state, and of the characteristic parameters of the size distribution of the material to be ground at the outlet of the mill at the stationary state. Discrete model number II. Parameters: I=20, J=30, m=3, n=2, V_F =0.5, V_B=0.1, L_{max}=2000.

Recirculation, Delay	Ks	Time required for reaching the stationary state	The average size of the material leaving the mill	The dispersion of the material leaving the mill
No	$0.5 \cdot 10^{-7}$	82	855.15	334.47
Yes, d=3	$0.5 \cdot 10^{-7}$	86	840.42	323.00
Yes, d=5	$0.5 \cdot 10^{-7}$	86	840.55	323.09
Yes, d=10	$0.5 \cdot 10^{-7}$	87	840.18	323.22

behaviour of the grinding system with and without classification and recirculation. In these simulation runs we used process and kinetic parameters given in Figs 1-4.

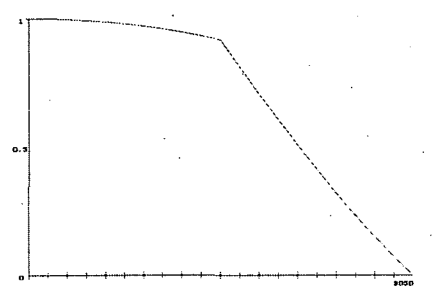
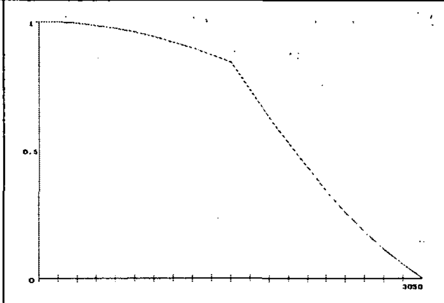
When grinding occurs without classification and partial recirculation of the product, the stationary state is reached at the t=76^{th} unit of the simulation time. In the case of applying classification and recirculation, however, the stationary state was reached at the 1153th unit of simulation time. The oversize distributions of the material to be ground at the outlet of the mill are shown in Figs 1-4 at t=10 and t=20 units of time. Figs 1-2 refer to grinding without recirculation. Comparing Figs 1 and 3, as well as Figs 2 and 4, it is seen that after a few units of time from the beginning of the process the oversize distribution functions are very similar to each other in both cases with or without recirculation. The effects of time delay and recirculation are hardly visible.





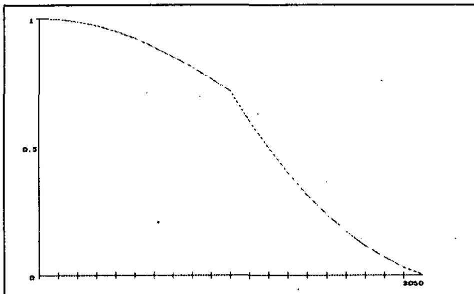
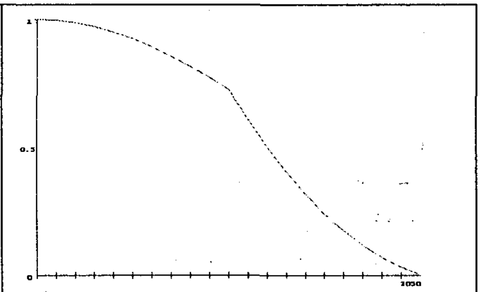
Average size: 2123, Dispersion: 534	Average size: 1995, Dispersion: 599

Figs 1-2. The oversize distributions of the particulate product at the outlet of the mill after t=10, t=20 units of time, respectively. Continuous grinding without recirculation. Discrete model number I. Parameters: I=20, J=20, m=2, n=2, V_F=0.5, V_B=0.1, L_{max}=3050, K_s= $9 \cdot 10^{-9}$.

	
Average size: 2123, Dispersion: 545	Average size: 1995, Dispersion: 599

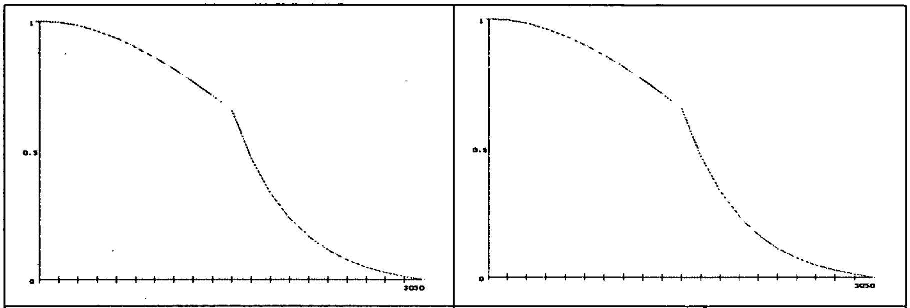
Figs 3-4. The oversize distributions of the particulate product at the outlet of the mill after t=10, t=20 units of time, respectively. Continuous grinding without recirculation. Discrete model number I. Parameters: I=20, J=20, m=2, n=2, V_F=0.5, V_B=0.1, L_{max}=3050, K_s= $9{\cdot}10^{-9}$, d=4.

Let us now see the oversize distribution functions at the 40^{th} unit of simulation time. Fig.5 refers to grinding without classification and recirculation, while Fig.6 refers to grinding with that. Here, we already see some differences between the two types of grinding mode. When the mill operates with recirculation the amount of particles belonging to the large particle size intervals is larger than in the opposite case. The oversize distribution function, shown in Fig.5, is similar to the oversize distribution function in stationary state, which is shown in Fig.10.

	
Average size: 1790, Dispersion: 644	Average size: 1800, Dispersion: 642
Fig.5. The oversize distribution function shown at the outlet of the mill after t=40 units of time. Continuous grinding without recirculation.	Fig.6. The oversize distribution function shown at the outlet of the mill after t=40 units of time. Continuous grinding with recirculation.

In this example, the stationary state is reached after rather long time when grinding is carried out with recirculation. The composition is changed very slowly.

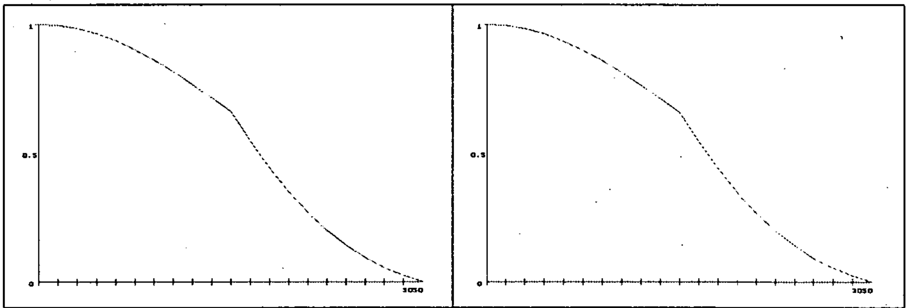
The oversize distribution functions are almost the same at t=600th, at t=900th and in the stationary state, as these are shown in Figs 7-8 and in Fig.12.



Average size: 1604, Dispersion: 582	Average size: 1603, Dispersion: 582

Figs 7-8. The oversize distribution functions shown at the outlet of the mill after t=600 and t=900 units of time, respectively. Continuous grinding with recirculation.

Close to the stationary state, the time evolution of the oversize distribution functions becomes very slow as it is shown in Figs 9-10 and in Figs 11-12.

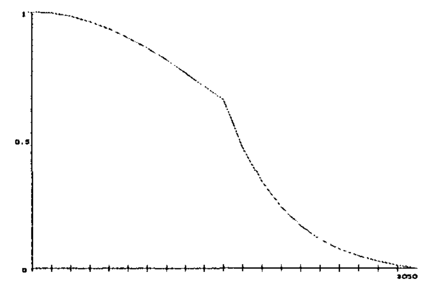


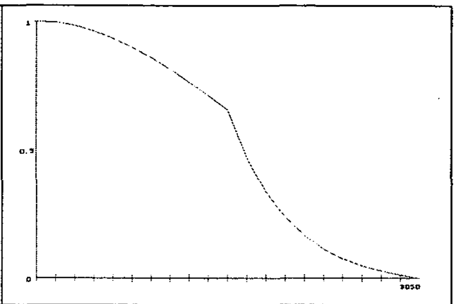
Average size: 1704, Dispersion: 654	Average size: 1700, Dispersion: 655

Figs 9-10. The oversize distribution functions shown at the outlet of the mill after t=66 and t=76 units of time, respectively. Continuous grinding without recirculation.

Figs 10 and 12 show that the stationary states are different in this case. When grinding was performed with classification and recirculation, the average size of the product was reduced with 5.7%. The time required for reaching the stationary state increased significantly due to the recirculation.

Let us now see what transient and stationary processes can be observed inside the mill. Since, by making the discretization of the partial integro-differential



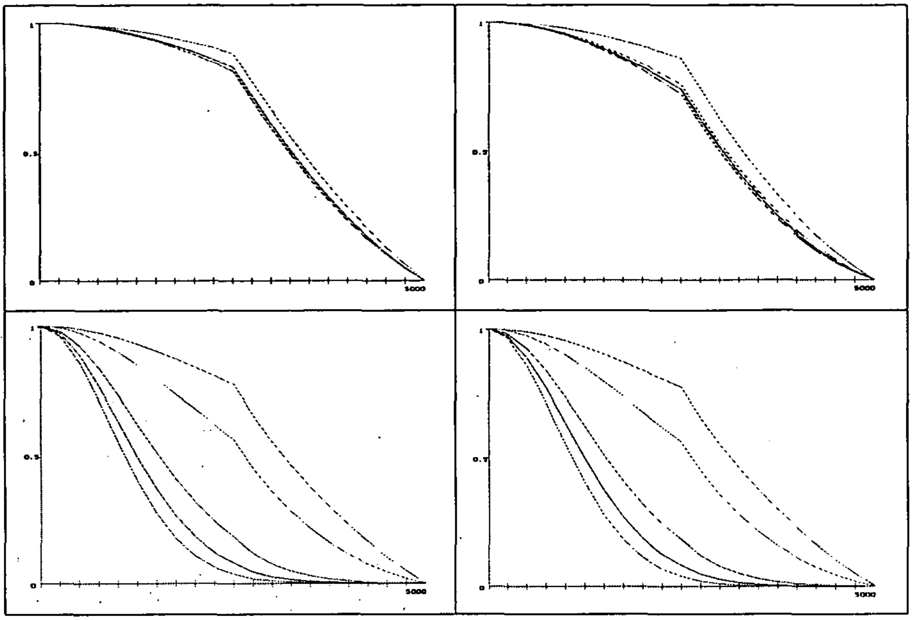


Average size: 1603, Dispersion: 581	Average size: 1603, Dispersion: 581

Figs 11-12. The oversize distributions shown at the outlet of the mill after t=1143 and t=1153 units of time, respectively. Continuous grinding with recirculation.

equation (1), we subdivided the length of the mill into 20 equal subintervals, subsequently called those columns, processes at different places inside the mill can be monitored as processes in the columns. The oversize distribution functions of the first, third, tenth, fifteenth and twentieth columns of the mill are shown in Figs 13-16, respectively, at $t=5^{th}$, $t=8^{th}$, $t=150^{th}$, $t=287^{th}$ units of simulation time by using the discrete model number I. It is seen that the stationary state is reached at $t=287^{th}$ units of time. Figs 9 and 10 show that after a few units of time the oversize distribution functions in the columns in question are very similar to each other since the initial loading of the mill and the size distribution of the fed material were chosen to be monodisperse. In stationary states, the size distribution of the material close to the inlet of the mill is quite different from that which is observed inside of the mill and near the outlet. The effects of the feed are seen only close to the inlet. The size distributions of the columns change only hardly approaching the stationary state. This is shown in Figs 15 and 16. In this case stationary state is reached very slowly.

The model number I, as it was defined before, describes such processes in the grinding system in which the total mass flow inside the mill, due to the dependence of the amount of the recycled material on the classifying device, may be changed. As a consequence, we can observe changes of the total mass flow in any column of the mill. The total mass flow of the particles in the first, tenth and last columns of the mill are shown in Fig.17 as a function of time. Due to the action of classification and recirculation, the total mass flow begins to increase in the first column of the mill after d=4 units of time Naturally in this case the time delay was 4 discrete time units. It is seen in Fig.17 that as the impulse of the total mass flow is shifted towards the outlet of the mill it is dispersed and the peak becomes smaller and smaller, i.e. the mixing process is smoothing the impulse. After reaching the new stationary state, the mass flow becomes equal and constant in time in each column of the mill, but this occurs at higher loading value.



Figs 13-16. The oversize distribution functions shown in the first, third, tenth, fifteenth and twentieth columns of the mill at $t=5^{th}$, $t=8^{th}$, $t=150^{th}$ and $t=287^{th}$ units of simulation time. Continuous grinding with recirculation. Discrete model number I. Parameters: $I=20$, $J=20$, $m=2$, $n=2$, $V_F=0.3$, $V_B=0.1$, $L_{max}=5000$, $K_s=10^{-8}$, $d=4$.

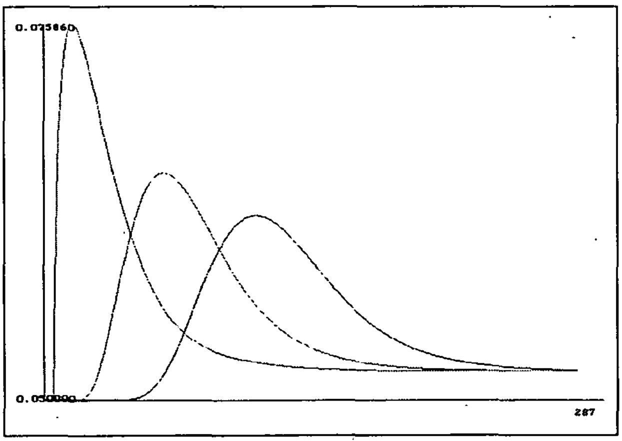


Fig.17. Variation of the total mass flow as a function of time in the first, tenth and last column of the mill, respectively. Continuous grinding with recirculation. Discrete model number I. Parameters: $I=20$, $J=20$, $m=2$, $n=2$, $V_F=0.3$, $V_B=0.1$, $L_{max}=5000$, $K_s=10^{-8}$, $d=4$.

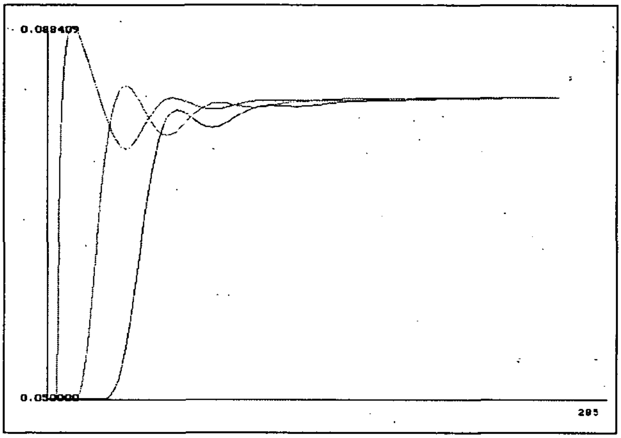


Fig.18. Variation of the total mass flow as a function of time in the first, tenth and last column of the mill, respectively. Continuous grinding with recirculation. Discrete model number I. Parameters: I=20, J=20, m=2, n=2, V_F =0.3, V_B=0.1, L_{max}=4400, K_s=10^{-8}, d=4.

In some case of simulation runs, damped oscillations of the total mass flow were detected as it is shown in Figs 18 and 19. In Fig.18, for instance, oscillations of the total mass flow in the first, tenth and last column of the mill are presented. In this case, in principle, oscillations become damped entirely after three decreasing characteristic peaks, and the mill reaches a stable stationary state at t=285^{th} units of simulation time.

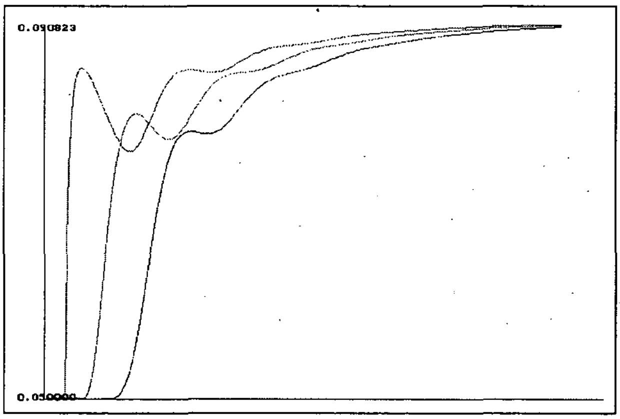


Fig.19. Variation of the total mass flow as a function of time in the first, tenth and last column of the mill, respectively. Continuous grinding with recirculation. Discrete model number I. Parameters: I=20, J=20, m=2, n=2, V_F =0.3, V_B=0.1, L_{max}=4000, K_s=10^{-8}, d=4.

Another example is shown in Fig.19, which is quiet different from the processes seen in Fig.18. In this case, the total mass flow exhibits oscillations with increasing peaks, although the amplitudes here are decreased, too, but after damping the oscillations the mass flow remains increasing monotonically.

In this case, the classifying device, because of the insufficient grinding efficiency of the mill, returns increasing amount of material to be ground again, thus, as a consequence, the load of the mill may increase above a given limit. It is an overloading phenomenon of the mill, and such grinding system is considered unstable.

5 Conclusions

Computer models and programs were elaborated for studying the stationary state processes of continuous grinding systems working with and without classification and partial recirculation of the product. The final form of the model was expressed as a set of recursive equations. The successive solution of the set of equations converges to the stationary state of the system.

The computer models developed are suitable for resolve a number of problems origin from the practice. For example, the problem of producing particulate product having prescribed average particle size with given constrains on the dispersion of the particle size distribution is a common one in process and mineral industry. By means of the newly developed programs it is possible to simulate the process in order to find the main properties and parameters of the grinding system which produces products satisfying the requirements of the end-users by efficient working of the grinding mill. These programs can also be used for estimating the kinetic parameters of the breakage processes, as well as for identifying the process parameters and conditions of the grinding devices and systems.

The efficiency of the grinding devices usually depends also on total mass flow and size distribution of the raw material fed into the system. The models and programs presented in the paper allow examining these effects, too. By simulating the transient processes caused by changes in the feed or recycled flows, the times required to reach the stationary states, as well as the conditions leading to overloading of the mill can be analysed and predicted, making possible to set the correct conditions of operation of the grinding systems in.

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