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Modelling a Sender-Receiver System

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Abstract

In this paper we present a sender-receiver system with an unlimited buffer modelled by a jumping Petri net, and then we prove some properties of the system.

Keywords: parallel/distributed systems, Petri nets, jumping Petri nets, modelling, verification.

1 Introduction and Preliminaries

A Petri net is a mathematical model used for the specification and the analysis of parallel/distributed systems. An introduction about Petri nets can be found in [4].

One formal analysis method for Petri nets is that of place and transition invariants, which were first introduced in [3]. Place and transition invariants are useful to prove dynamic properties, like reachability, boundedness, home state, liveness and fairness properties.

It is well-known that the behaviour of some distributed systems cannot be adequately modelled by classical Petri nets. Many extensions which increase the computational and expressive power of Petri nets have been thus introduced. One direction has led to various modifications of the firing rule of nets. One of these extensions is that of jumping Petri net, introduced in [5].

Let us briefly recall the basic notions and notations concerning Petri nets and jumping Petri nets in order to give the reader the necessary prerequisites for the understanding of this paper (for details the reader is referred to [1], [4], [2]). Mainly, we will follow [2], [5].

A *Place/Transition net*, shortly *Petri net*, (finite, with infinite capacities), is a 4-tuple $\Sigma = (S, T, F, W)$, where S and T are two finite non-empty sets (of *places* and *transitions*, resp.), with $S \cap T = \emptyset$, $F \subseteq (S \times T) \cup (T \times S)$ is the *flow relation* and $W : (S \times T) \cup (T \times S) \to \mathbb{N}$ is the *weight function* of Σ verifying $W(x, y) = 0$ iff $(x, y) \notin F$.

A *marking* of a Petri net Σ is a function $M : S \to \mathbb{N}$; it will be sometimes identified with a $|S|$-dimensional vector. The operations and relations on vectors are componentwise defined. $\mathbb{N}^S$ denotes the set of all markings of Σ.

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A *marked Petri net* is a pair $\gamma = (\Sigma, M_0)$, where Σ is a Petri net and M_0, called the *initial marking* of γ, is a marking of Σ.

Let Σ be a Petri net, $t \in T$ and $w \in T^*$. The functions $t^-, t^+ : S \to \mathbb{N}$ and $\Delta t, \Delta w : S \to \mathbb{Z}$ are defined by: $t^-(s) = W(s,t)$, $t^+(s) = W(t,s)$, $\Delta t(s) = t^+(s) - t^-(s)$, and

$$\Delta w(s) = \begin{cases} 0, & \text{if } w = \lambda \\ \sum_{i=1}^{n} \Delta t_i(s), & \text{if } w = t_1 t_2 \ldots t_n \ (n \geq 1) \end{cases}, \text{ for all } s \in S.$$

The sequential behaviour of a Petri net Σ is given by the so-called *firing rule*, which consists of

- the *enabling rule*: a transition t is *enabled* at a marking M in Σ (or t is *fireable* from M), abbreviated $M[t\rangle_\Sigma$, iff $t^- \leq M$;
- the *computing rule*: if $M[t\rangle_\Sigma$, then t may *occur* yielding a new marking M', abbreviated $M[t\rangle_\Sigma M'$, defined by $M' = M + \Delta t$.

In fact, any transition t of Σ establishes a binary relation on $\mathbb{N}^S$, denoted by $[t\rangle_\Sigma$ and given by: $M[t\rangle_\Sigma M'$ iff $t^- \leq M$ and $M' = M + \Delta t$.

If $t_1, t_2, \ldots, t_n$ $(n \geq 1)$ are transitions of Σ, $[t_1 t_2 \ldots t_n\rangle_\Sigma$ will denote the classical product of the relations $[t_1\rangle_\Sigma, \ldots, [t_n\rangle_\Sigma$. Moreover, the relation $[\lambda\rangle_\Sigma$ is considered, by defining $[\lambda\rangle_\Sigma = \{(M,M) | M \in \mathbb{N}^S\}$.

Let $\gamma = (\Sigma, M_0)$ be a marked Petri net, and $M \in \mathbb{N}^S$. The word $w \in T^*$ is called a *transition sequence* from M in Σ if there exists a marking M' of Σ such that $M[w\rangle_\Sigma M'$. Moreover, the marking M' is called *reachable* from M in Σ. The set of all reachable markings from M_0 is called *the reachability set* of γ, and it is denoted by $[M_0\rangle_\gamma$.

A place $s \in S$ is *bounded* if there exists $k \in \mathbb{N}$ such that $M(s) \leq k$, for all $M \in [M_0\rangle_\gamma$. The net γ is *bounded* if all its places are bounded.

A transition $t \in T$ is *live* if for any reachable marking $M \in [M_0\rangle_\gamma$, there exists a marking M' reachable from M such that t is fireable from M'. The net γ is *live* if all its transitions are live.

In order to be able to define the notion of the incidence matrix for a Petri net $\Sigma = (S,T,F,W)$, it is necessary to have a total ordering of the sets S and T. Without loss of generality, it will be assumed that, if these sets are of the form

$$S = \{s_1, \ldots, s_m\}, \text{ and } T = \{t_1, \ldots, t_n\},$$

then they are totally ordered by the natural order on the indexes of the elements:

$$S: \ s_1 < \ldots < s_m, \text{ and } T: \ t_1 < \ldots < t_n.$$

The *incidence matrix* of a Petri net Σ is the $m \times n$-dimensional matrix I_Σ defined by

$$I_\Sigma(i,j) = \Delta t_j(s_i), \ \forall\, 1 \leq i \leq m, \ \forall\, 1 \leq j \leq n.$$

The notion of incidence matrix is extended also to marked Petri nets (Σ, M_0) through the unmarked underlying net Σ.

An *S-invariant* (or *place invariant*) of Σ is any m-dimensional vector J of integer numbers which satisfies the equation $J \cdot I_\Sigma = \mathbf{0}$.

The characterization theorem of S-invariants says that, if J is an S-invariant of a marked Petri net $\gamma = (\Sigma, M_0)$, then the relation

$$J \cdot M = J \cdot M_0$$

holds for any $M \in [M_0\rangle_\gamma$. In other words, this theorem says that any S-invariant of γ gives the weights for the places of a subnet of γ in which the tokens are preserved (through these weights).

Jumping Petri nets ([5]) are an extension of Petri nets, which allows them to do "spontaneous jumps" from one marking to another one (this is similar to λ-moves in automata theory).

A *jumping Petri net* is a pair $\gamma = (\Sigma, R)$, where Σ is a Petri net and R is a binary relation on the set of markings of Σ (i.e. $R \subseteq \mathbb{N}^S \times \mathbb{N}^S$), called the *set of (spontaneous) jumps* of γ.

Let $\gamma = (\Sigma, R)$ be a jumping Petri net. The pairs $(M, M') \in R$ are referred to as *jumps* of γ. Σ is called the *underlying Petri net* of γ. A *marking* of γ is any marking of its underlying Petri net. If γ has finitely many jumps (i.e. R is finite), then γ is called a *finite jumping Petri net.*

For any jump $r = (M, M') \in R$, the function $\Delta r : S \to \mathbb{Z}$ is defined by $\Delta r(s) = M'(s) - M(s)$, for all $s \in S$. If the set of jumps R has finitely many variations (i.e. the set $\Delta R = \{\Delta r \mid r \in R\}$ is finite), then γ is called a *Δ-finite jumping Petri net.*

A *marked jumping Petri net* is defined similarly as a marked Petri net, by changing "Σ" into "Σ, R".

Pictorially, a jumping Petri net will be represented as a classical net and, moreover, the relation R will be separately listed.

The behaviour of a jumping Petri net γ is given by the *j-firing rule,* which consists of

- the *j-enabling rule*: a transition t is *j-enabled* at a marking M (in γ), abbreviated $M[t\rangle_{\gamma,j}$, iff there exists a marking M_1 such that $MR^*M_1[t\rangle_\Sigma$ (R^* being the reflexive and transitive closure of R);
- the *j-computing rule*: if $M[t\rangle_{\gamma,j}$, then the marking M' is *j-produced* by occurring t at M, abbreviated $M[t\rangle_{\gamma,j}M'$, iff there exists two markings M_1, M_2 such that $MR^*M_1[t\rangle_\Sigma M_2R^*M'$.

The notions of *transition j-sequence* and *j-reachable marking* are defined similarly as for Petri nets (the relation $[\lambda\rangle_{\gamma,j}$ is defined by $[\lambda\rangle_{\gamma,j} = R^*$). The *set of all j-reachable markings* of a marked jumping Petri net γ is denoted by $[M_0\rangle_{\gamma,j}$ (M_0 being the initial marking of γ).

All other notions from Petri nets (i.e. boundedness, liveness, etc.) are defined for jumping Petri nets similarly as for Petri nets, by considering the notion of *j-reachability* instead of *reachability* from Petri nets.

Some jumps of a marked jumping Petri net may be never used. Thus a marked jumping Petri net $\gamma = (\Sigma, R, M_0)$ is called *R-reduced* iff, for any jump $(M, M') \in R$, $M \neq M'$ and $M \in [M_0\rangle_{\gamma,j}$.

The notion of place invariants for Δ-finite jumping Petri nets, and results regarding them, can be found in [6]. We will briefly present this notion.

As in the case of Petri nets, in order to be able to define the notion of the incidence matrix for a Δ-finite jumping Petri net $\gamma = (\Sigma, R)$, where $\Sigma = (S, T, F, W)$ is the underlying Petri net of γ, it is necessary to have a total ordering of the sets S, T and ΔR. Without loss of generality, it will be assumed that, if these sets are of the form

$$S = \{s_1, \ldots, s_m\},\ T = \{t_1, \ldots, t_n\}, \text{ and } \Delta R = \{\Delta r_1, \ldots, \Delta r_p\},$$

then they are totally ordered by the natural order on the indexes of the elements:

$$S:\ s_1 < \ldots < s_m,\ T:\ t_1 < \ldots < t_n, \text{ and } \Delta R:\ \Delta r_1 < \ldots < \Delta r_p.$$

The *incidence matrix* of a Δ-finite jumping Petri net $\gamma = (\Sigma, R)$ is the $m \times (n+p)$-dimensional matrix I_γ defined by

$$I_\gamma(i,j) = \begin{cases} I_\Sigma(i,j) & ,\forall\ 1 \le j \le n \\ I_R(i, j-n) & ,\forall\ n+1 \le j \le n+p \end{cases} ,\forall\ 1 \le i \le m,$$

where I_Σ is the incidence matrix of the underlying Petri net of γ and I_R is the $p \times n$-dimensional matrix given by

$$I_R(i,j) = \Delta r_j(s_i),\ \forall\ 1 \le i \le m,\ \forall\ 1 \le j \le p\ .$$

The notion of incidence matrix is extended also to marked Δ-finite jumping Petri nets (Σ, R, M_0) through the unmarked underlying net (Σ, R).

An *S-invariant* (or *place invariant*) of γ is any m- dimensional vector J of integer numbers which satisfies the equation $J \cdot I_\gamma = \mathbf{0}$. The S-invariant $J > \mathbf{0}$ is called *minimal* if there exists no S-invariant J' such that $\mathbf{0} < J' < J$.

The characterization theorem of S-invariants ([6]) says that, if J is an S-invariant of a marked Δ-finite jumping Petri net $\gamma = (\Sigma, R, M_0)$, then the relation

$$J \cdot M = J \cdot M_0$$

holds for any $M \in [M_0\rangle_{\gamma,j}$. As in the case of Petri nets, the meaning of this theorem is that any S-invariant of γ gives the weights for the places of a subnet of γ in which the tokens are preserved (through these weights).

The paper is organized as follows. Section 2 presents an example of a sender-receiver system modelled by a jumping Petri net, and section 3 presents the verification of the system properties using the place invariant method. Section 4 concludes this paper.

2 Sender-receiver with unlimited buffer

This section presents an example of using jumping Petri nets to model and analyse real systems.

Let us consider a system consisting of a sender (producer) and a receiver (consumer). The sender produces and sends messages to the receiver, one by one, through an asynchoronous channel (a buffer with unlimited capacity for storing messages). The receiver receives and consumes, one by one, the messages from channel. Moreover, the sender can take a break at any moment, but we impose the restriction that the receiver can enter his inactive state only if the sender is inactive and there is no message pending in the channel.

The same system, but with a limited buffer, was modelled by a Petri net in [4]. Unfortunately, this system with an unlimited buffer cannot be modelled by a Petri net because zero tests of a location with infinite capacity cannot be simulated by Petri nets (a proof of this fact can be found in [2], where a similar system with an unlimited buffer is modelled by a Petri net with inhibitor arcs).

A modelling of this system by a finite jumping Petri net $\gamma = (\Sigma, R, M_0)$ is presented in Figure 1, with the following interpretation of places:

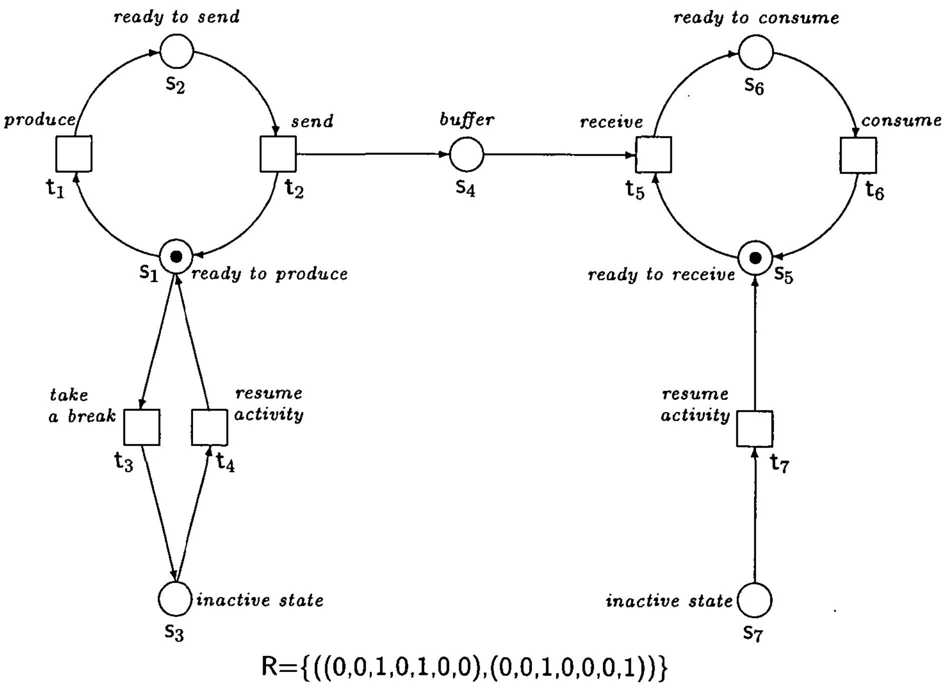

Figure 1: Sender-receiver system with unlimited buffer

- s_1 marked = the sender is ready to produce a message or to take a break;
- s_2 marked = the sender is ready to send the last produced message;
- s_3 marked = the sender is inactive (in a break);
- s_4 = the unlimited buffer for storing messages;
- s_5 marked = the receiver is ready to receive a message or to take a break;
- s_6 marked = the receiver is ready to consume the last received message;
- s_7 marked = the receiver is inactive (in a break).

The interpretations of transition firings are the following:

- t_1 = the sender produces a message;
- t_2 = the sender sends a message;
- t_3 = the sender becomes inactive (takes a break);
- t_4 = the sender resumes his activity;
- t_5 = the receiver receives a message;
- t_6 = the receiver consumes a message;
- t_7 = the receiver resumes his activity.

The entering of the receiver in his inactive state, possible only when the sender is inactive and there are no messages in the buffer, is modelled by the jump of this net, which occurs from the marking $M' = (0,0,1,0,1,0,0)$ to the marking $M''=(0,0,1,0,0,0,1)$.

We say that the sender-receiver system with an unlimited buffer is *modelled correctly*, if it has the following properties:

(P_1) At any moment, the sender is in one of the states "ready to produce", "ready to send" or "inactive";

(P_2) At any moment, the receiver is in one of the states "ready to receive", "ready to consume" or "inactive";

(P_3) The buffer can contain any number of messages;

(P_4) The receiver can enter his inactive state only if the sender is in his inactive state and there are no messages in the buffer;

(P_5) The system is live, i.e. it will never reach a deadlock state.

In the next section we will show how the verification of these properties can be done.

3 Verification of system properties

Using S-invariants, we prove in this section the correctness of our modelling.

Theorem 3.1. *The jumping Petri net from Figure 1 models correctly the sender-receiver system with unlimited buffer.*

Proof. Let $\gamma = (\Sigma, R, M_0)$ be the finite jumping Petri net from Figure 1. It is easy to verify that the vectors

$$J_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix},$$

are S-invariants. Moreover, these are the only minimal S-invariants of γ.

Let $M \in [M_0\rangle_{\gamma,j}$ be an arbitrary j-reachable marking of γ. Using the S-invariant J_1 and the characterization theorem of S-invariants, we find that

$$(*) \quad M(s_1) + M(s_2) + M(s_3) = 1,$$

which proves $(\mathbf{P_1})$. Similarly, using J_2 we obtain that

$$(**) \quad M(s_5) + M(s_6) + M(s_7) = 1,$$

which proves $(\mathbf{P_2})$.

In order to prove $(\mathbf{P_3})$, let us notice the following fact. Given any $k \in \mathbb{N}$, by firing the transition sequence $w = (t_1 t_2)^k$ at the marking M_0, a new marking $M \in [M_0\rangle_{\gamma,j}$ will be produced, with $M(s_4) = k$ and $M(s) = M_0(s)$ for all other places of the net. This means that the buffer can contain any number of messages.

In order to prove $(\mathbf{P_4})$, let us notice that, if M is an arbitrary j-reachable marking in which the receiver is inactive (i.e. $M(s_7) = 1$), then M can be reached only by the occurence of the jump of the net γ (because $M \notin [M_0\rangle_\Sigma$, i.e. the marking M is not reachable in the underlying Petri net of γ). It is obviously that the jump of γ can occur only if the sender is inactive and the message channel is empty.

For proving the net γ is live, i.e. it never reaches a deadlock state, we will show that at any j-reachable marking $M \in [M_0\rangle_{\gamma,j}$ there exists at least one transition of γ which is fireable at M. Indeed, from the equality $(*)$ follows that either the transitions t_1 and t_3 are fireable at M, if $M(s_1) = 1$, or the transition t_2 is fireable at M, if $M(s_2) = 1$, or the transition t_4 is fireable at M, if $M(s_3) = 1$. Therefore, the net from Figure 1 is live, which proves $(\mathbf{P_5})$.

This concludes the proof of the system properties. □

Let us remark that from the last argument from above follows also that the sender is live (i.e. the net γ w.r.t. the set of transitions $\{t_1, t_2, t_3, t_4\}$ is live).

Moreover, the receiver (i.e. the net γ w.r.t. the set $\{t_5, t_6, t_7\}$) is not live, but "almost live", i.e. it never deadlocks excepting the case when the sender is active and the message channel is empty. Indeed, from the equality $(**)$ follows that the only possible cases are the following ones:

i) the transition t_7 is fireable at M, if $M(s_7) = 1$;

ii) the transition t_6 is fireable at M, if $M(s_6) = 1$;

iii) the transition t_5 is fireable at M, if $M(s_5) = 1$ and $M(s_4) > 0$;

iv) the transition t_7 is j-fireable at M (after occuring first the jump of the net at M), if $M(s_5) = 1$, $M(s_4) = 0$ and $M(s_3) = 1$;

v) the case $M(s_5) = 1$, $M(s_4) = 0$ and $M(s_3) = 0$, i.e. the case in which the sender is active ("ready to produce" or "ready to send") and the message channel is empty, is the only case when the receiver has no directly possible action, but only after an action of the sender (either the producing of a message, or the sending of a message, or the entering of the sender in his inactive state).

4 Conclusion

In this paper we have modelled a sender-receiver system with an unlimited buffer by a finite jumping Petri net, and we have proved the correctness of our modelling by using S-invariants.

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