Acta Cybernetica **18** (2008) 367–377.

On the Ambiguity of Reconstructing *hv*-Convex Binary Matrices with Decomposable Configurations*

Péter Balázs†

Abstract

Reconstructing binary matrices from their row, column, diagonal, and antidiagonal sums (also called projections) plays a central role in discrete tomography. One of the main difficulties in this task is that in certain cases the projections do not uniquely determine the binary matrix. This can yield an extremely large number of (sometimes very different) solutions. This ambiguity can be reduced by having some prior knowledge about the matrix to be reconstructed. The main challenge here is to find classes of binary matrices where ambiguity is drastically reduced or even completely eliminated. The goal of this paper is to study the class of *hv*-convex matrices which have decomposable configurations from the viewpoint of ambiguity. First, we give a negative result in the case of three projections. Then, we present a heuristic for the reconstruction using four projections and analyze its performance in quality and running time.

Keywords: discrete tomography; *hv*-convex binary matrix; decomposable configuration; reconstruction algorithm

The reconstruction of binary matrices from their projections is a basic problem in discrete tomography. Binary matrices can represent two-dimensional cross-sections of an object made (or consisting) of homogeneous material, while one can think of projections as the numerical results of measuring the density of the object in the given cross-section along certain directions. Reconstruction algorithms have a wide area of applications in non-destructive testing, biplane angiography, crystallography, radiology, image processing, and so on. For a detailed overview of the main problems and applications of discrete tomography the reader is referred to [11, 12]. For practical reasons the projections in most cases can be taken only from few (usually at most four) directions. This often leads to ambiguous reconstruction, i.e., the reconstructed matrix can be quite dissimilar to the original one which is inappropriate for applications [1, 14]. One commonly used technique to reduce ambiguity is to use some a priori information of the matrix to be reconstructed. In this paper we investigate the problem of ambiguity if the matrix to

*This work was supported by OTKA grant T048476.

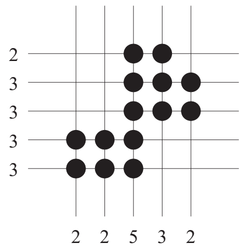
†Department of Image Processing and Computer Graphics, University of Szeged, 6720 Szeged, Árpád tér 2., Hungary, E-mail: pbalazs@inf.u-szeged.hu

be reconstructed is *hv*-convex and its components form a so-called *decomposable configuration*. First, we give a construction to prove that the use of only three projections is not sufficient to eliminate ambiguity, that is, for some inputs there can be exponentially many *hv*-convex decomposable binary matrices having the same horizontal, vertical and diagonal projections. In the case of four projections we are facing the following problem. Although all the *hv*-convex decomposable binary matrices with the given four projections can be reconstructed in polynomial time [2] this class is not explicitly defined. In more detail, one criterion for decomposability is that the components of the binary matrix are uniquely reconstructible from their horizontal and vertical projections. However, when reconstructing *hv*-convex matrices with the algorithm of [2] it cannot be decided in advance whether this criterion is satisfied. If so then, clearly, the algorithm gives correct solutions. However, in some cases the algorithm gives a solution even if the above criterion is not fulfilled, i.e, if one or more of the components are not uniquely determined by the horizontal and vertical projections. We conduct experiments to investigate whether the above criterion is often implicitly satisfied. Since components of an *hv*-convex binary matrix are necessarily *hv*-convex polyominoes, we investigate the possibility that an *hv*-convex polyomino is uniquely determined by its horizontal and vertical projections. We also study how the knowledge of a component's third projection affects ambiguity, and based on the observations we develop a fast and accurate reconstruction heuristic for the class of *hv*-convex binary matrices with decomposable configurations.

This contribution is structured as follows. First, the necessary preliminaries are introduced in Section 1. In Section 2 we show that in the class of *hv*-convex decomposables there could be a large number of ambiguous reconstructions, if we use only three projections. In Section 3 we extend the method published in [2] for reconstructing *hv*-convex binary matrices with decomposable configurations even if it is not guaranteed that the components are uniquely determined by the horizontal and vertical projections, and analyze the performance of the developed algorithm. Finally, in Section 4 we discuss our results.

1 Preliminaries

Discrete sets (the finite subsets of the two-dimensional integer lattice) are highly important in discrete tomography. A discrete set with the smallest containing discrete rectangle (SCDR) of size $m \times n$ can be represented by a binary matrix $F = (f_{ij})_{m\times n}$ where the 1s in the matrix are representing that the corresponding element of the 2D lattice belongs to the discrete set (see Fig. 1). Based on this correspondence, in the sequel, when we are using the term discrete set we always mean the set of positions of F having value 1. Analogously, the *size* of the discrete set is defined by the size of its SCDR (or equivalently, the size of its representing matrix F). To avoid confusion we stress that the size of the discrete set is not the number of its elements (see, again, the discrete set of Fig. 1 which is of size 5×5 but has 14 elements). The *horizontal* and *vertical projections* of F are the vectors



$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Figure 1: An *hv*-convex polyomino and the corresponding binary matrix. The elements of the discrete set are marked with black dots. The projections of the polyomino are the vectors $H = (2, 3, 3, 3, 3)$, $V = (2, 2, 5, 3, 2)$, $D = (0, 2, 3, 2, 1, 1, 2, 2, 1)$, and $A = (0, 0, 1, 3, 4, 4, 2, 0, 0)$.

$\mathcal{H}(F) = H = (h_1, \ldots, h_m)$, and $\mathcal{V}(F) = V = (v_1, \ldots, v_n)$, respectively, where

$$h_i = \sum_{j=1}^{n} f_{ij} \quad (i = 1, \ldots, m) \quad \text{and} \quad v_j = \sum_{i=1}^{m} f_{ij} \quad (j = 1, \ldots, n) \; . \tag{1}$$

Similarly, the *diagonal* and *antidiagonal projections* of F are defined by $\mathcal{D}(F) = D = (d_1, \ldots, d_{m+n-1})$, and $\mathcal{A}(F) = A = (a_1, \ldots, a_{m+n-1})$, respectively, where

$$d_k = \sum_{i+(n-j)=k} f_{ij} \quad \text{and} \quad a_k = \sum_{i+j=k+1} f_{ij} \quad (k = 1, \ldots, m+n-1) \; . \tag{2}$$

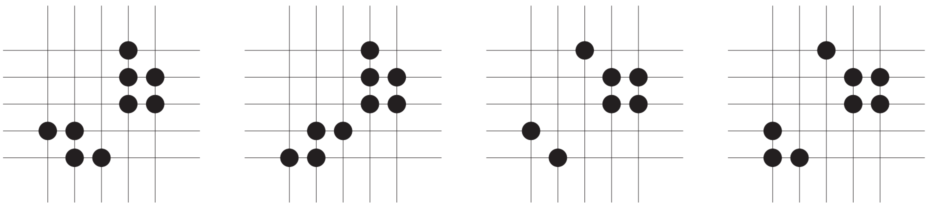
Two positions $P = (p_1, p_2)$ and $Q = (q_1, q_2)$ in a discrete set are said to be *4-adjacent* if $|p_1 - q_1| + |p_2 - q_2| = 1$. The positions P and Q are *4-connected* if there is a sequence of distinct positions $P_0 = P, \ldots, P_k = Q$ in the discrete set F such that P_l is 4-adjacent to P_{l-1} for each $l = 1, \ldots, k$. A discrete set F is *4-connected* if any two points in F are 4-connected. The 4-connected discrete set is also called *polyomino*. A maximal 4-connected subset of a discrete set F is called a *component* of F. In particular, every polyomino consists of a single component. The discrete set F is *hv-convex* if all the rows and columns of F are 4-connected (see Fig. 1).

Given an ordered pair of binary matrices (C, D) we say that we get the binary matrix F by *NorthWest-gluing* (or shortly, NW-gluing) C to D if

$$F = \begin{pmatrix} C & \mathbf{0} \\ \mathbf{0} & D \end{pmatrix} \; . \tag{3}$$

If C is a polyomino then we say that C is the *NW-component of F*. NE-, SE-, SW-gluings and -components are defined similarly. A discrete set F consisting of $k \geq 2$ components is *decomposable* if all of the following properties are fulfilled

(α) the components of F are uniquely reconstructible from their horizontal and vertical projections in polynomial time,



(a) (b) (c) (d)

Figure 2: (a)-(c) Undecomposable and (d) decomposable hv-convex discrete sets.

(β) the sets of the row and column indices of the components' SCDRs are disjoint,

(γ) if $k > 2$ then we get F by gluing a single component to a decomposable discrete set consisting of $k-1$ components using one of the four gluing operators.

If F satisfies properties (β) and (γ) but not necessarily property (α) then we say that the components of F are in a *decomposable configuration*. Obviously, every hv-convex set satisfies property (β). The discrete set in Fig. 1 is undecomposable since it consists of only one component. Figures 2a and 2b show a situation where the components are in a decomposable configuration but property (α) does not hold since the bottom left components of both sets have the same horizontal and vertical projections. The discrete set in Fig. 2c does not satisfy property (γ) while the set shown in Fig. 2d is decomposable.

The reconstruction task aims to find a discrete set F such that $\mathcal{H}(F) = H$ and $\mathcal{V}(F) = V$ for given vectors H and V (in the case of four projections two more vectors D and A are also given, i.e., $\mathcal{D}(F) = D$ and $\mathcal{A}(F) = A$ must also hold). Not any pair (or 4-tuple) of vectors are projections of a discrete set (see, e.g., [16] for a necessary and sufficient condition in the case of two projections). However, in some cases there can be several (and also very dissimilar) solutions with the same projections (see, e.g, [14]). This latter feature of the reconstruction is the so-called ambiguity, a problem one tries to avoid during the reconstruction. One of the most frequently used techniques to reduce ambiguity is to suppose that the set to be reconstructed belongs to a certain class of discrete sets having some geometrical properties. There are classes of discrete sets where ambiguity is completely eliminated (see [3, 4, 8]). Furthermore, for certain classes it was shown that only a polynomial number of discrete sets with the same projections can belong to the given class [2, 5]. Finally, for some classes it is known that ambiguity in those classes can be exponentially large [4, 8, 10]. In this paper we are going to investigate the problem of ambiguity in the class of hv-convex discrete sets having decomposable configurations.

2 Three Projections: A Negative Result

In [2] it was shown that every *hv*-convex decomposable discrete set having the same horizontal, vertical, diagonal, and antidiagonal projections can be reconstructed in polynomial time. Clearly, this also means that the number of solutions is polynomial, too. However, the question was left open whether the use of four projections is necessary to achieve this result. The following theorem gives an answer to this question.

Theorem 1. *For some vectors H, V, and D there can be exponentially many hv-convex decomposable binary matrices with the same horizontal, vertical, and diagonal projections H, V, D, respectively.*

Proof. Consider the following matrices

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad M' = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{4}$$

Clearly, M and M' are decomposable and have the same horizontal, vertical, and diagonal projections. Now, for a given $k \geq 1$ and for any $S \subseteq \{1, \ldots, k\}$ let the matrix X_k^S be defined as follows

$$X_k^S = \begin{pmatrix} M_1 & & & \\ & M_2 & & \\ & & \ddots & \\ & & & M_k \end{pmatrix} \quad \text{where} \quad M_i = \begin{cases} M & \text{if } i \in S \\ M' & \text{if } i \notin S \end{cases} \tag{5}$$

for every $i = 1, \ldots, k$. The matrices defined by (5) are, certainly, *hv*-convex and decomposable and have the same horizontal, vertical, and diagonal projections. S can be any subset of $\{1, \ldots, k\}$ which gives 2^k matrices with the described properties. □

As a consequence of this theorem we get

Corollary 1. *If there is an algorithm that reconstructs all the hv-convex decomposable binary matrices with the horizontal, vertical, and diagonal projections H, V, and D, respectively, then there are some vectors H, V, and D for which the time complexity of the algorithm is exponential.*

Remark 1. *Naturally, we get the same results replacing the diagonal projections with the antidiagonal projections.*

3 Four Projections: A Reconstruction Heuristic

The reconstruction of a discrete set from four projections is NP-hard [10]. Furthermore, the number of *hv*-convex discrete sets having the same four projections can be extremely large. This can be shown, e.g., in a similar way as in the proof of Theorem 1 but using the matrices

$$M = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad M' = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} . \tag{6}$$

However, as we mentioned in Section 2 if we assume that the set is hv-convex and decomposable then every discrete set with the given four projections can be found in polynomial time yielding a polynomial number of solutions. Before going further we describe a somewhat modified version of the algorithm given in [2].

Algorithm 1.

Input: Four vectors, $H \in \mathbb{N}^m$, $V \in \mathbb{N}^n$, $D, A \in \mathbb{N}^{m+n-1}$.
Output: The binary matrix F with $\mathcal{H}(F) = H$, $\mathcal{V}(F) = V$, $\mathcal{D}(F) = D$, and $\mathcal{A}(F) = A$ or the message "no solution".

1: $F = \emptyset$;
2: try to find the bottom right corner (i, j) of a component in the NW-corner;
3: **if** (Step 2 succeed) **then** reconstruct all the hv-convex polyominoes with horizontal and vertical projections $(h_1, \ldots, h_i)$ and $(v_1, \ldots, v_j)$, respectively;
else goto Step 5;
4: select randomly a polyomino P with $\mathcal{A}(P) = (a_1, \ldots, a_{i+j-1})$ from the candidates generated in Step 3;
5: **if** (no component) **then** try to decompose a component in the NE-corner using the vectors H, V, and D similarly as in Steps 2, 3, and 4;
if (no component) **then** try to decompose a component in the SE-corner using the vectors H, V, and A similarly as in Steps 2, 3, and 4;
if (no component) **then** try to decompose a component in the SW-corner using the vectors H, V, and D similarly as in Steps 2, 3, and 4;
if (no component) **then goto** Step 6;
else { update H, V, D, and A according to the projections of P;
$F = F \cup P$;
goto Step 2; }
6: try to reconstruct the last component and update the vectors;
7: **if** $(H = V = D = A = 0)$ **then return** F **else** FAIL (no solution);

The algorithm in its original form reconstructs hv-convex decomposable discrete sets with the given four projections in polynomial time by decomposing a component (which is an hv-convex polyomino) in each step of the main loop. Since in that class property (α) is satisfied the components are reconstructed from their horizontal and vertical projections uniquely. However, when reconstructing a component we always have a third projection which is not used directly for reconstruction but only for testing whether the reconstructed polyomino in the given corner has

Table 1: The number of *hv*-convex polyominoes in the test data sets that are not uniquely determined by two, three, and four projections.

Size $n \times n$	H, V	H, V, D	H, V, A	H, V, D, A
4×4	1393	40	52	18
5×5	1442	33	36	16
7×7	967	13	8	2
10×10	586	4	6	1
20×20	312	2	1	1
40×40	210	1	0	0
60×60	162	1	0	0
80×80	148	0	0	0
100×100	171	0	0	0

the proper (diagonal or antidiagonal) third projection. Interestingly, in some cases the algorithm also gives a solution even if one or more of the components are not uniquely determined by the horizontal and vertical projections, i.e., if there are ambiguous reconstructions for some of the components (see [2] for further details). Algorithm 1 exploits this feature of the original algorithm to serve as a heuristic for the broader class of *hv*-convexes with decomposable configurations. The idea of our reconstruction heuristic is that we try to eliminate ambiguity by using directly the third projection in the reconstruction. In Step 3 of Algorithm 1 we reconstruct all candidates for a component from the horizontal and vertical projections. Then, in Step 4 we choose one of them that has the proper third projection. Note that if the discrete set to be reconstructed satisfies property (α), too, then Steps 3 and 4 together yield the original form of the algorithm presented in [2].

We have conducted experiments for investigating how ambiguity of the components (which are *hv*-convex polyominoes) can affect the performance of Algorithm 1. Using the methods given in [13], we have generated *hv*-convex polyominoes with different sizes sampled from a uniform random distribution. Each test data set consisted of 5000 discrete sets with the same size. The second column of Table 1 represents the number of polyominoes in the test data sets that have ambiguous solutions when only two projections are used to reconstruct them. Note that unless the size of the polyomino is small ambiguity occurs in only 3-6% of the cases (these results of this column are essentially the same that were established independently by a similar investigation in [7]). This means that if the components of an *hv*-convex discrete set form a decomposable configuration and they are relatively big then it is very likely that the set will be decomposable, i.e., it will satisfy property (α), too. Clearly, the more components the discrete set has, the less likely it is that all the components are uniquely determined by the horizontal and vertical projections. Moreover, if the set has small components then ambiguity can occur more likely – possibly causing the algorithm to fail.

The accuracy of Algorithm 1 depends on whether the third projection can effectively eliminate the ambiguity. The third and fourth columns of Table 1 represent the number of polyominoes in the test data sets that are not uniquely determined by three projections (clearly, due to symmetry the two columns have nearly the same entries). These results show that if the size of the polyomino is greater than 3×3 then ambiguity occurs in less than 1% of the cases, and it reduces drastically as the size of the set increases. Again, the more components the discrete set has the more likely ambiguity occurs (since it can occur in any component). If we have several candidates with the same three projections for a certain component then the only thing that affects the remaining part of the reconstruction is the fourth projection of the component. The fifth column of Table 1 shows that even in the cases if there are several candidates with the same three projections, it still has a small probability that the fourth projection of the chosen set will be the same as the true component's one, and thus the algorithm will not fail.

The computational cost of Algorithm 1 mostly depends on whether Step 3 can be performed fast. In this step we reconstruct hv-convex polyominoes from their horizontal and vertical projections. Several algorithms have been developed for solving this problem (see [6] for a comparison of them). However, all of them can find only one of the solutions in polynomial time. Since the number of hv-convex polyominoes with the same horizontal and vertical projections can be exponentially large [9] executing Step 3 in some cases can take an exponential time. Fortunately, in average case this task can be performed in a few hundredths seconds even on a PC with a processor of only 533 MHz [6].

Based on the observations that all the hv-convex polyominoes having the same two projections can be found quite fast, and the number of ambiguous cases is very small if three projections are used to reconstruct them, we expect our newly developed heuristic to reconstruct hv-convex discrete sets having decomposable configurations (i.e., if property (α) might not hold) fast and in most cases accurate. In order to support this claim we have conducted some experiments. We have generated 5 data sets, each of them contained 1000 hv-convex sets with decomposable configurations having k components of size $n \times n$ for some fixed k and n. The generation method was the following. Again, using the methods given in [13], we have generated a sequence of k hv-convex polyominoes of size $n \times n$ sampled from a uniform random distribution. Then, we have generated a random sequence of the elements NW, NE, SE, and SW. If the discrete set to be generated consisted of k components then the length of the sequence was $k - 1$, and it represented the way and order of how the k components should be glued together. For the 5 test data sets we have chosen the parameters k and n as follows:

- Test 1: $k = 10$, $n = 5$;
- Test 2: $k = 20$, $n = 5$;
- Test 3: $k = 30$, $n = 5$;
- Test 4: $k = 10$, $n = 10$;
- Test 5: $k = 20$, $n = 10$.

Table 2: Accuracy and average running time of Algorithm 1 on the test data sets.

Test	#correct sol.	#incorrect sol.	#no sol.	time (s)
Test 1	939	14	47	0.600
Test 2	891	27	82	0.847
Test 3	851	41	108	2.322
Test 4	998	0	2	0.660
Test 5	994	0	6	5.676

For example, Test 1 contained 1000 discrete sets of size 50×50, and each of them had 10 components of size 5×5. The reconstruction heuristic was implemented in C++ and the test run on an Intel Pentium IV 3.2GHz with 1GB RAM under Debian GNU/Linux 3.1. Table 2 shows the average running times, and the number of correct and incorrect solutions for the 5 test data sets. From the entries of Table 2 we can deduct that the number of incorrect solutions increases as the number of components increases. But we should mention here that in the first three tests an inaccurate reconstruction differed from the original set just in one component, and just in 8 positions. More precisely, the original and the reconstructed components always formed a pair like

$$M = \begin{pmatrix} 0 & 0 & X & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ X & 1 & 1 & 1 & X \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & X & 0 & 0 \end{pmatrix} \quad \text{and} \quad M' = \begin{pmatrix} 0 & 1 & X & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ X & 1 & 1 & 1 & X \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & X & 1 & 0 \end{pmatrix}, \tag{7}$$

where the positions marked by X can take arbitrary values and are the same in M and M'. Moreover, according to Table 1 if the set has components of size 10×10 (or bigger) then the algorithm can reconstruct the set in almost all cases, accurately. In fact, in Tests 4 and 5 we did not find inaccurate reconstructions (although it has a small probability that an inaccurate solution will be reconstructed). Evidently, the larger the set is the more time is needed to reconstruct it, but even for the biggest sets of Test 5 the average running time of the algorithm is very fast. We should also add that the implementation was not optimised on time. Summarizing this we can say that Algorithm 1 has really good performance in both quality and running time.

4 Conclusions and Further Research

We have studied the problem of ambiguity for reconstructing *hv*-convex discrete sets which have decomposable configurations. We have shown that if only the horizontal, vertical, and one of the diagonal projections of the set to be reconstructed are known then the number of solutions with the same projections can be extremely large. It is an open question whether in this case a reconstruction algorithm can be given to find a solution in polynomial time. If we assume that all four projections of the discrete set are given then the reconstruction of *hv*-convex decomposables can be achieved in polynomial time. We extended this reconstruction algorithm to the more general case when the components of the *hv*-convex set are not necessarily uniquely determined by their two projections but still form a decomposable configuration. Although the extended algorithm in some cases uses exponential time to reconstruct a solution, experimental results show that the average running time of the enhanced algorithm is very fast. The algorithm in some cases does not find a solution (or not the original one). However, our investigation also shows that this happens very rarely (especially if the set has components of size larger than 5×5).

In [15] an algorithm is presented to reconstruct *hv*-convex discrete sets from horizontal and vertical projections. The worst case time complexity of this algorithm is exponential, too [17]. This algorithm is suitable to reconstruct *hv*-convex sets from four projections as well. In this case one should simply reconstruct every *hv*-convex discrete set which have the same given horizontal and vertical projections and then keep only the solutions that also have the proper diagonal and antidiagonal projections. It would be interesting to compare the average performance of this algorithm to our newly developed one. However, the generation of general *hv*-convex sets using uniform random distributions is an unsolved problem, therefore no method is known by which the comparison on the whole set of *hv*-convexes could be done. In the future we want to search for subclasses of the class of *hv*-convexes whose elements can be generated using uniform random distributions, and which are general enough for doing significant comparisons.

Acknowledgments

The author of this paper would like to thank the anonymous reviewer for his suggestions which considerably improved the quality of the paper.

References

[1] A. Alpers, P. Gritzmann, L. Thorens, Stability and instability in discrete tomography, *Proceedings of Digital and Image Geometry, Lecture Notes in Comput. Sci.*, **2243** (2001) 175–186.

[2] P. Balázs, A decomposition technique for reconstructing discrete sets from four projections, *Image and Vision Computing*, accepted.

[3] P. Balázs, Reconstruction of discrete sets from four projections: strong decomposability, *Elec. Notes in Discrete Math.*, **20** (2005) 329–345.

[4] P. Balázs, The number of line-convex directed polyominoes having the same orthogonal projections, *Proceedings of the 13th International Conference on*

Discrete Geometry for Computer Imagery DGCI 2006, Lecture Notes in Comput. Sci., **4245** (2006) 77–85.

[5] P. Balázs, E. Balogh, A. Kuba, Reconstruction of 8-connected but not 4-connected *hv*-convex discrete sets, *Disc. App. Math.*, **147** (2005) 149–168.

[6] E. Balogh, A. Kuba, Cs. Dévényi, A. Del Lungo, Comparison of algorithms for reconstructing *hv*-convex discrete sets, *Lin. Alg. Appl.* **339** (2001) 23–35.

[7] S. Brunetti, A. Del Lungo, F. Del Ristoro, A. Kuba, M. Nivat, Reconstruction of 4- and 8-connected convex discrete sets from row and column projections, *Lin. Alg. Appl.* **339** (2001) 37–57.

[8] A. Del Lungo, Polyominoes defined by two vectors, *Theoret. Comput. Sci.* **127** (1994) 187–198.

[9] A. Del Lungo, M. Nivat, R. Pinzani, The number of convex polyominoes recostructible from their orthogonal projections, *Discrete Math.* **157** (1996) 65–78.

[10] R.J. Gardner, P. Gritzmann, Uniqueness and complexity in discrete tomography, In [11] (1999) 85–113.

[11] G.T. Herman, A. Kuba (Eds.), *Discrete Tomography: Foundations, Algorithms and Applications*, Birkhäuser, Boston, 1999.

[12] G.T. Herman, A. Kuba (Eds.), *Advances in Discrete Tomography and Its Applications*, Birkhäuser, Boston, 2007.

[13] W. Hochstättler, M. Loebl, C. Moll, Generating convex polyominoes at random, *Discrete Math.* **153** (1996) 165–176.

[14] T.Y. Kong, G.T. Herman, Tomographic equivalence and switching operations, In [11] (1999) 59–84.

[15] A. Kuba, The reconstruction of two-directionally connected binary patterns from their two orthogonal projections, *Comp. Vision, Graphics, and Image Proc.* **27** (1984) 249–265.

[16] H.J. Ryser, Combinatorial properties of matrices of zeros and ones, *Canad. J. Math.* **9** (1957) 371–377.

[17] G.W. Woeginger, The reconstruction of polyominoes from their orthogonal projections, *Inform. Process. Lett.* **77** (2001) 225–229.