Acta Cybernetica **19** (2009) 5–15.

Backprojection Reconstruction Algorithm Using Order Statistic Filters In Breast Tomosynthesis

László Csernetics

Abstract

Breast cancer is the most common cancer type and one of the leading cause of death among women. It has been recognized over the years that preventing the disease is the most powerful weapon, and the implementation of screening mammography has had significantly reduced the death rate. However, it is also proven that conventional mammography does not detect approximately 30% of breast cancers. Inventing new imaging technologies for the earlier detection of breast cancer is vital and is in the center of many ongoing studies. There are several new techniques using different imaging modalities that are under investigation. The most promising is the breast tomosynthesis, an advanced x-ray application that addresses the problem of structure superimposition, one of the major deficiencies of 2D mammography, by reconstructing a range of slices providing additional 3-dimensional information of the breasts.

Our goal is to investigate and develop reconstruction algorithms that fit into the new mathematical model of tomosynthesis used in mammography. In this paper we show a backprojection reconstruction technique that is especially well-suited for the problem in question. This algorithm is capable to produce contrast-enhanced slices of the breast by taking only the projections that most probably hold the "important" information of the targeted lesions, ignoring part of the projections. This statistical approach also offers a good noise management performance, as a fortunate side-effect. After discussing the algorithm we publish the results of the comparison of this technique with other popular methods of the algorithm-family. We also look out the strict boundaries of the work done suggesting improvements of the reconstruction algorithm.

Keywords: breast tomosynthesis, mammography, breast cancer, reconstruction algorithm

1 Introduction

Today, breast cancer is the most common type of cancer among women, about 1.3 million new cases will be diagnosed in the near future annually worldwide, and about 465,000 will die from this disease, according to the American Cancer Society.

DOI: 10.14232/actacyb.19.1.2009.2

With other words, in the future one of every 8 women is expected to be diagnosed with breast cancer at some point in her lifetime. On the other hand, the death rate of this type of cancer is steadily dropping since the 90's, due to the successful breast imaging techniques introduced and used in the practice of screening exams, especially in the developed countries. Although the death rate has been halved over the years, it has been proven that the commonly used mammographic techniques are still imperfect tools. In fact, around 30% of breast cancers are missed in the first year of presence, because of the lack of sensitivity and specificity of the conventional mammography ([6, 8, 9]).

The limitations of mammography are originated from the 2D characteristics of the technique. The healthy (functional- and fatty-) tissues and the potential abnormalities are superimposed on the 2D image, making it difficult to detect and classify the different structures of the breast. The overlapped normal structures could imitate abnormalities, resulting in additional, but unnecessary examinations of the patient. In other cases, the abnormal structures could be surrounded and hidden within the normal tissues of the breast, causing false-negative detections, and increasing the risk of developing breast cancer (refer to [1] for more detailed comparison of mammographic techniques).

Breast tomosynthesis, a new 3D mammographic technique that is still under development, is targeting exactly this deficiency of conventional mammography. By reconstructing a range of 2-dimensional horizontal slices, it provides much more visual information of the breast. Moreover, by using new reconstruction algorithms that are fine-tuned for breast tomosynthesis, the contrast of the structures could be improved, helping the physicians to establish more precise diagnosis. This is also the subject of our research, and we are going to show such an algorithm in this paper.

2 Digital breast tomosynthesis

Digital breast tomosynthesis is a technique on the way from the conventional mammography to the computerized tomography ([10, 11]). The procedure of image acquisition and also the device itself are very similar to the currently applied technology, as it has been developed from 2D mammography. Therefore, the patients are already familiar with it, what is more, this new technique is even more comfortable. It preserves the advantages of mammography, most importantly the low radiation dose used during an examination. The only difference becomes evident with the quality of the produced images, as well as the efficiency of the method. It is clear, that this technique is about to replace today's imaging modalities of screening examinations.

Tomosynthesis is an image processing technique for generating 2-dimensional slices of the imaged organ, while all the information that is needed for such a reconstruction is not necessarily available. Usually, only a narrow angular range of image acquisition is given, and only a limited number of projections can be taken. The tomographic techniques commonly used in medical imaging are not facing such constraints, and in those cases the complete 3D data of the imaged volume can be reconstructed. This is not possible in the case of tomosynthesis, where only a series of slices can be reconstructed, containing an additional depth information, that becomes clearly visible when the reader of the mammogram loops through the slices. This is what tomosynthesis really offers for the physicians, it virtually eliminates the structure superimposition presented on the conventional mammograms.

3 Projection geometry

The story of breast tomosynthesis is quite new, the first studies have been published in the 90's, and the system has still not been introduced into the practical usage. In spite of this fact, there are already several independent research groups, including the pioneers of the field from the Massachusetts General Hospital (MGH), and the research team of the Duke University Medical Center, working for the same goal. One of the main differences we can find between the actually ongoing studies is the geometrical model implemented with the system.

Generally, we can speak about two main models whose basic concepts are the same, and they differ only in a few minor geometrical aspects, including the number of projections taken and the angular range of x-ray tube rotation. In our work, we are referring the model used by the group at the MGH, also published in [2, 7, 8]. Its schematic representation can be seen in Figure 1.

In tomosynthesis, the x-ray detector is stationary, while the x-ray tube is rotating during the image acquisition. Before an examination starts, the breast has to be fixated with the compression paddles of the device, located above the detector. This step is needed only for the immobilization of the breast, no real compression like in conventional mammography is applied (this is why this method is much more comfortable). The imaging process itself takes only 7 seconds, and 11 low radiation dose images are taken. During this time, only the x-ray tube is moving on an arc of $50°$, starting from the tilted position of $+25°$, making $5°$ angle increments for every image. The distance between the x-ray tube and the detector is 66cm, where the axis of rotation is at 44.3cm from the source, and at 21.7cm from the detector. The sixth image is taken from the position where the electron rays are hitting the detector at a right angle (like the CC view of the conventional mammography). The total radiation dose of such an examination is about 1.5 times more than in the 2D mammography.

Using the projections produced by the above mentioned way, it is possible to reconstruct the 3D dataset of the imaged volume. However, the limited input of the model states a difficult image reconstruction problem. It is obvious, that the commonly used basic reconstruction algorithms will not provide the desired results, because of the high error ratio indicated by the very few number of projections, as well as the narrow angular range. An algorithm using some kind of *a-priori* knowledge, that is capable to deal with the error and noise presented in the input images has to be employed. As an additional aim, it would be nice to extract the

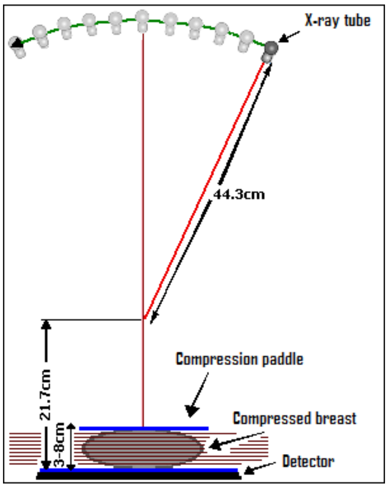

Figure 1: The schematic representation of the used projection geometry. The x-ray tube is in the starting position.

targeted structures on the results. In the second part of the paper, we are going to show an algorithm that is especially well-suited to breast tomosynthesis, and also able to produce contrast-enhanced reconstructed slices of the imaged breast.

4 Order statistic based reconstruction algorithm

Generally speaking, reconstructing an object from its projections is the inverse transformation of the projection generation process itself. Reconstruction algorithms can be divided into two main groups, regarding the way of approaching the result (more about reconstruction algorithms can be found in [4]). The MLEM (Maximum-Likelihood Expectation Maximization), as an iterative algorithm, represents one of the most successful methods of doing the reconstruction in breast tomosynthesis, however, it is very complex, and the reconstruction of a high quality image set usually takes several hours. On the other hand, the non-iterative algorithms are much more simple, since they are simply projecting back the projections into the volume to be reconstructed. This is why these methods are the most widely accepted techniques in the field. In our work, we are investigating a variant of the conventional filtered backprojection algorithm, that has already been employed in the tomosynthesis studies for angiography, decades ago. Beside simply adopting the technique onto the field of breast tomosynthesis, we are going to prove its validness for the problem in question. We have to emphasize that this is a heuristic approach. At some points the formulation is only a crude approximation, rather than a mathematically precise derivation of the algorithm.

Before going into the details of the algorithm, let us take a closer look at the already mentioned transformations. The projection, known also as the Radon-transform, has the following form:

$$p^{\phi}(s_1, s_2) = \int_{-\infty}^{\infty} f^{\phi}(s_1, t, s_2) dt = [Rf^{\phi}](s_1, s_2), \tag{1}$$

where p^{ϕ} denotes the 2D projection taken under the angle ϕ, and f^{ϕ} denotes the 3D object in the rotated coordinate system of the device. The backprojection operator can be written up as the formal adjoint of the Radon-transform:

$$[R^* p](x, y, z) = \tilde{f}(x, y, z) = \int_{-\theta}^{\theta} p^{\phi}(x \cos\phi + y \sin\phi, z) d\phi. \tag{2}$$

However, this is not the inversion of the Radon-transform, it gives a blurred recovery of the original image. In fact, applying the backprojector operator on R results in the convolution of the original image function with a blurring function hl:

$$[R^* R f] = \tilde{f} = (f * hl). \tag{3}$$

The image could be fully recovered only by applying the appropriate inverse filtering on $\tilde{f}$.

We can notice, that in our case the third dimension is not affecting the calculations, since we have a tilted geometry, with only one degree of freedom (tilting around the z axis). After a normalization with the number of projections, we get the appropriate averages of the backprojected values in every point of the volume. Discretizing the formula in (2) will finally lead us to the basic equation of the simple backprojection algorithm:

$$V(X) = \tilde{f}(x, y, z) = \frac{1}{k} \cdot \sum_{n=1}^{k} P_n(\bar{X}), \tag{4}$$

where $V(X)$ denotes the value of the voxel belonging to the position determined by (x, y, z), k denotes the number of projections, and $P_n(\bar{X})$ denotes the pixel value of the nth projection at the position calculated from the above mentioned coordinate triplet.

This simple backprojection algorithm is mathematically clear, straightforward to implement, and has low computational costs. However, since the projection values are equally distributed on the ray-path belonging to a certain projection position, and the reconstructed values are simply the averages of the backprojected values, the result of this method is not satisfactory. Applying filters in the frequency domain of the projections is a very common way of improving the quality of the reconstruction. Still, in our case this is not enough to achieve the best possible result, because of the limited input data. To make this method even more suitable for breast tomosynthesis, the basic conception of the algorithm has to be modified as well.

In the middle of 80's, researchers from Hamburg have been introduced a new non-linear method, as they called, the extreme-value reconstruction ([3, 5]). The main idea was to replace the averaging operator used in (4) with a minimum-type operator:

$$V^*(X) = \min_{n=1}^{k}\{P_n(\bar{X})\}. \tag{5}$$

This way the reconstructed voxels were holding the minimum of the backprojected values instead of their average, which is convenient in the case of angiography, since the expected values are approximately 0 everywhere, except the vessels, due to the injected contrast material (Digital Subtraction Angiography).

When we examine a few mammograms, we can see, that the situation is very similar to the angiography. The fatty tissues are appearing as very dark areas, the functional and other healthy tissues are brighter, while the abnormal structures are very bright, or white regions. Since we are primarily interested in the correct reconstruction of the abnormal structures potentially presented in the breast, after an appropriate preprocessing of the projections, we can apply the above mentioned extreme-value reconstruction. On the other hand, it is also obvious that there is a relation between the number of projections used in the reconstruction of a voxel and the noise-characteristics of the reconstruction algorithm itself. While averaging the projections in a reconstruction step has a noise reducing effect, using only one projection will end up in a much more noisy result, because of the extreme-value selection. Thus, combining the two operators could lead us to a more optimal result. As a first approach, we can construct an averaging operator that averages all, but the K largest projections (to preserve the advantages of the minimum-type operator). But, ignoring also the minimal (generally, the L smallest) value could improve the noise-management of the algorithm. This way we finally end up with the so called order statistic filter (OS-filter, or L-filter):

$$V^{**}(X) = \frac{1}{(k-L-K)} \cdot \sum_{n=L+1}^{k-K} P_n(\bar{X}), \tag{6}$$

where the projections are ordered by their value at the position $\bar{X}$ every time a voxel is being reconstructed:

$$P_{min}(\bar{X}) \le \cdots \le \underbrace{P_{L+1}(\bar{X}) \le \cdots \le P_{k-K}(\bar{X})} \le \cdots \le P_{max}(\bar{X}). \tag{7}$$

In the case, when there are more projections with the same value in the series above, the randomly permutated order of these values should be used in order to avoid some projections to be preferred to others. We have to emphasize, that we are going to employ a minimum-type operator, so the value of L should be chosen to be small, while the K should be larger. Regarding to the observations done so far, we suggest the selection of $\{L = 2, K = 4\}$.

Note the relationship between the equations covered above. We can write up these operators with one common formula:

$$\widetilde{V}(X) = \frac{\sum_{n=1}^{k} \omega_n P_n(\bar{X})}{\sum_{n=1}^{k} \omega_n}, \tag{8}$$

where the selection of the ω_n weights determines the type of the operator described with the formula (e.g. $\omega_n = 1$ for every n is the averaging operator). The appropriate selection of the weights depends on the *a-priori* knowledge associated with the problem. Particularly, the filter introduced above is defined by the following weight selection strategy:

$$\omega_n = \begin{cases} 1 & \text{if } P_{L+1}(\bar{X}) \leq P_n(\bar{X}) \leq P_{k-K}(\bar{X}) \\ 0 & \text{otherwise} \end{cases} , \tag{9}$$

where the indexing of the projections is in accordance with (7).

From this point we are going to discuss the reconstruction algorithm. Before starting with the first step, a pre-processing of the input projection images is needed. The purpose of this step is to correct the errors caused by the ray scattering effect during the image acquisition, as well as to normalize the projections against the geometrical distortion presented in the images caused by the varying ray-path lengths within the imaged breast under the varying angles of incidence. After this correction, every pixel of the projections will hold the average attenuation value measured on the corresponding ray-path. We can also make a background subtraction following the idea from digital subtraction angiography, assuming that the fatty tissues are appearing as homogeneous, dark areas on the images. It is important, however, to perform this kind of correction globally, to preserve the images normalized. Since the algorithm is based on projection ordering, it is very sensitive to intensity-level differences presented between the projections. This is also the reason for leaving the projections unfiltered for the reconstruction.

In the first step of the algorithm we are doing a reconstruction using the operator in (6) as the basis of the backprojection algorithm. As it was expected, the reconstructed horizontal slices show reduced error rate caused by the so called out-of-plane artifacts, the side-effect of the averaging operator. Also, the noise-management has been improved comparing to the extreme-value reconstruction. But, in spite of the seemingly good results, our algorithm is completely incorrect in the mathematical point of view. When we analyze the result of the reconstruction, it becomes obvious that the re-projection consistency constraint won't be satisfied. This criterion states against a reconstruction algorithm to produce a result that is consistent with the projection data, i.e. if a second projection is applied on the reconstructed data, the very same projections should be produced as in the case of the original object. Mathematically, since the projection values are representing the average attenuation values along the corresponding ray-paths, this means the following:

$$P_n(x) = \frac{1}{N} \cdot \sum_{i=1}^{N} R_i(\bar{x}), \tag{10}$$

where $P_n(x)$ is the value of nth projection at the detector-position x, N is the number of the reconstructed slices, and $R_i(\bar{x})$ is the value of the pixel $\bar{x}$ on ith slice. Of course, this would be satisfied in the case when the projection values are distributed equally on all of the reconstructed slices (like in the case of the averaging operator), but not in our case, when part of the projections is simply ignored during the reconstruction of each of the voxels.

As the second step of the algorithm, we are going to modify the original projections in such a way, that after doing a second order-statistic based reconstruction the re-projection consistency constraint will become satisfied. Let's first write up the equation in (10) in the following form (from this point we will denote the ith slice of the first and the final reconstruction with S_i and R_i, respectively):

$$P_n(x) = \frac{1}{N} \cdot \sum_{i \in \kappa} R_i(\bar{x}) + \frac{1}{N} \cdot \sum_{i \notin \kappa} R_i(\bar{x}), \tag{11}$$

where $i = 1, \ldots, N$, and $\kappa \subseteq \{1, \ldots, N\}$ is the set of slices where the $P_n(x)$ value has no contribution in the reconstruction of the pixel $\bar{x}$. The elements of the κ set is being collected during the first reconstruction step, for each pixel on every projection. Since the reconstructed values found at $\bar{x}$ on these slices are not affected by the projection value $P_n(x)$, we can accept the result of the first reconstruction at these positions. However, since the modification of the other projections could have an influence on these values, we are accepting these values only as the approximations of the final reconstruction values. Thus:

$$\sum_{i \in \kappa} R_i(\bar{x}) \approx \sum_{i \in \kappa} S_i(\bar{x}). \tag{12}$$

Rewriting (11) we get:

$$\sum_{i \notin \kappa} R_i(\bar{x}) \approx N \cdot \left(P_n(x) - \frac{1}{N} \cdot \sum_{i \in \kappa} S_i(\bar{x}) \right), \tag{13}$$

the approximate sum of the reconstructed values to which also the x pixel from the projection P_n had contribution during the first reconstruction. Since it is assumed that, after the pre-processing step, every projection value is representing the average attenuation, and the number of slices receiving contributions from $P_n(x)$ is N minus the cardinality of κ, we have to modify our projections in the following way:

$$\widetilde{P}_n(x) = \frac{N}{N - |\kappa|} \cdot \left(P_n(x) - \frac{1}{N} \cdot \sum_{i \in \kappa} S_i(\bar{x}) \right). \tag{14}$$

Now, that we have these enhanced projections, in the third step of the algorithm we are doing a final reconstruction of the horizontal slices. As in the previous case, the technique preserves the advantageous behavior of both the minimum-type and the averaging operators. Furthermore, the result is now mathematically correct, or, at least a mathematically correct approximation, regarding to the re-projection consistency constraint. As a fortunate side-effect of the projection modification, the contrast of the objects has been improved in the result, which was our additional criterion against the algorithm. This contrast enhancement could be explained by the characteristics of the κ set. Note, that a projection value and the cardinality of the corresponding κ set are linearly related, since the higher values are more probably ignored during the reconstruction. At the same time, it is obvious from (14) that the projections ignored in most of the cases will be modified in the highest degree. This way, in the final result, the smaller, but higher intensity objects will have an increased contrast. As we know, this is very well suited to the breast tomosynthesis, since we are interested in discovering of such kind of structures.

5 Experimental results

In our experiments we compared the order-statistic based backprojection algorithm with the algorithms employing the conventional averaging and the extreme-value operator. We ran the tests for ideal and noisy projections as well.

The experiments have been performed on an Intel P4 2.4GHz machine, with 1GB RAM memory, and Windows XP operation system on it. We have generated breast phantoms containing several objects of different sizes and placements, that are responsible for simulating the inner structure of the breast. 27 of them are bigger spheres filling out the volume, and representing the healthy mass of a breast. The other 7 spheres are smaller, but having more concentrated mass, thus, higher attenuation coefficient, and imitating different kind of abnormalities. 5 of these objects are very small, simulating calcium deposits in the breast. These calcifications could be the first signs of a malign abnormality, so it is important to find them as soon as possible. The other 2 bigger objects are imitating already developed abnormal structures embedded within the healthy tissues. The parameters of the algorithm have been set to $\{L = 2, K = 4\}$, and 21 horizontal slices of size 512x300 have been reconstructed with each algorithm, separately. The results can be seen in Figure 2.

Regarding to the time complexity of these algorithms, they are performing very similarly, as it could be expected. The average time needed to finish a complete reconstruction (including the two runs of the backprojection algorithm, as well as the projection modification step) was 153.15 seconds. This is acceptable comparing to the time consumption of an iterative algorithm.

Looking into the reconstructed images it can be seen that the algorithm using the averaging operator produced fairly the worst results. Although, the noise-management was quite good, the objects on the slices are low-contrasted, thus, this method is not convenient to detect the structures in question. On the other hand, the minimum operator was able to restore the shapes nicely, but in the noisy case it performed weakly. Eventually, the algorithm presented in this paper produced the best results, tolerating the noise and increasing the contrast of the objects at the same time.

Of course, regardless to the good results given, the algorithm has to be developed further in the future. We are supposing, that an iterative projection enhancing technique could improve the results, since the applied approximation during the derivation is converging toward satisfying the re-projection consistency constraint.

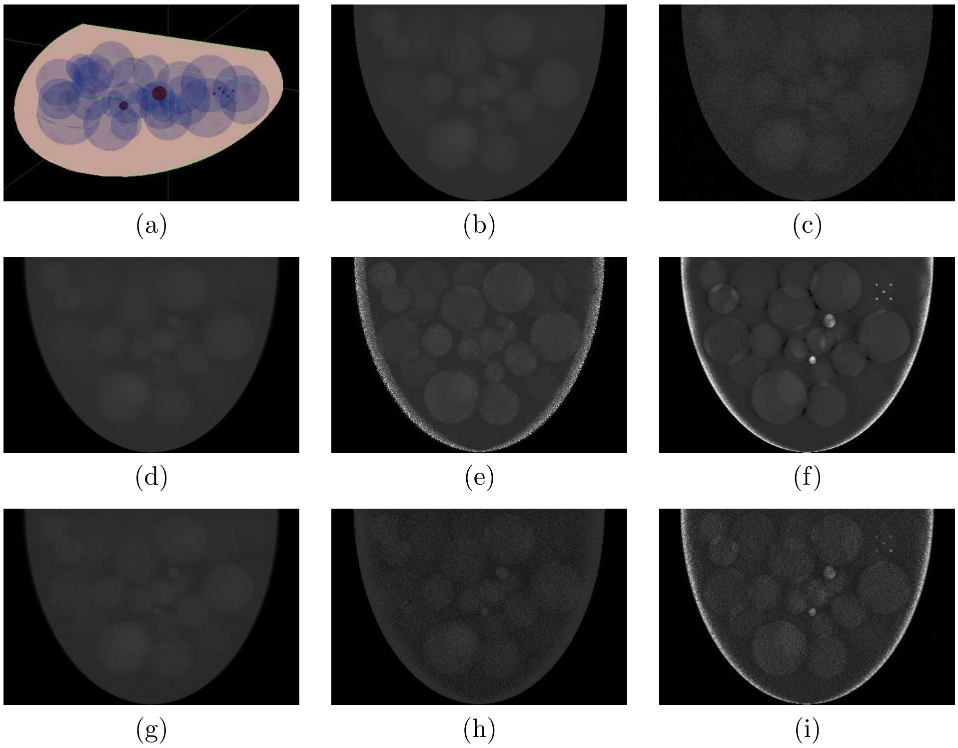

Figure 2: The top row presents the 3D breast phantom containing the 7 abnormalities with darker colors (a), the ideal (b), and the noisy 6th projection (c). The two rows below contains the central (11th) reconstructed slice in the ideal (d-f) and noisy (g-i) case: using the averaging operator (d, g), the minimum operator (e, h), and the order-statistic based operator (f, i), respectively. As it is clearly visible, this latter produced the most contrasted result, especially in the point of view of the targeted objects.

Also, more realistic breast phantoms, or even real data as input of the algorithm should be used in the experiments, to get more accurate results. Finally, the reconstructed slices themselves could be also improved by employing image processing techniques for image refinement.

Acknowledgements

The author would like to express thanks to his mentor, Dr. Attila Kuba, who supported this work with a great deal of help in research. Without his great experience and knowledge these results could not have been born. The author also received a considerable deal of support and inspiration to prepare this paper and keep on studying the research topic from his supervisor, Dr. Kálmán Palágyi. Finally, the author is thankful for the valuable comments to the reviewers of the paper.

References

[1] American Cancer Society: Mammograms and other breast imaging procedures. Last revised: 09/26/2008. Accessed August 2008.

[2] Csernetics, L.: Melltomoszintézis: 3D Mammográfia. Proc. 6th Conference of Hungarian Association for Image Processing and Pattern Recognition (2007) 9–19

[3] Haaker, P., Klotz, E., Koppe, R., Linde, R., Möller, H.: A new digital tomosynthesis method with less artifacts for angiography. Med. Phys. **12(4)** (1985) 431–436

[4] Herman, G.T.: Image reconstruction from projections. The fundamentals of computerized tomography. Academic Press, NY (1980) 1–40, 55–65, 90–147

[5] Klotz, E., Haaker, P., Koppe, R., Linde, R.: Three-dimensional angiography with digital flashing tomosynthesis. Adv. Imag. Proc. SPIE Proc. **804** (1987) 306–313

[6] Pisano, E.D., Gatsonis, C., Hendrick, E., Yaffe, M., Baum, J.K., Acharyya, S., Conant, E.F., Fajardo, L.L., Bassett, L., D'Orsi, C., Jong, R., Rebner, M.; Digital Mammographic Imaging Screening Trial (DMIST) Investigators Group: Diagnostic performance of digital versus film mammography for breast-cancer screening. N Engl J Med **353(17)** (2005) 1773–1783

[7] Rafferty, E.A.: Advances in imaging: Breast tomosynthesis. ASCO, Virtual Meeting (2004)

[8] Rafferty, E.A.: Breast tomosynthesis: Three-dimensional mammography. ASCO, 40th Annual Meeting (2004) 45–47

[9] Rafferty, E.A.: Tomosynthesis: New weapon in breast cancer fight. Decisions in Imaging Economics **17(4)** (2004)

[10] Smith, A.P.: Contrast enhanced breast tomosynthesis: A promising tool for identifying breast cancer. Hospital Imaging & Radiology Europe **1(3)** (2006)

[11] Wu, T., Rafferty, E.A., Kopans, D.B., Moore, R.H.: Contrast-enhanced breast tomosynthesis. DBT for better breast cancer diagnosis. RT Image **17(41)** (2004)