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Trace Simulation Semantics is not Finitely Based over BCCSP*

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Abstract

This note shows that the trace simulation preorder does not have a finite inequational basis over the language BCCSP. Indeed, no collection of sound inequations of bounded depth is ground-complete with respect to the trace simulation preorder over BCCSP even over a singleton set of actions.

Keywords: trace simulation preorder, complete axiomatizations, BCCSP

1 Introduction

The study of the equational theory of several algebraic structures has been one of the main research interests of the late Zoltán Ésik—see, for instance, the references [2, 3, 8, 9, 10, 13, 14, 16] for a small sample of his work in this area.

In the setting of process algebras, the study of complete axiomatizations of behavioural equivalences can be traced back to the early contributions of Hennessy and Milner [18], and Bergstra and Klop [7]. Since then, the investigation of the equational theory of various process algebras has been a major topic of research and Zoltán Ésik has contributed to this field in many ways—see, for instance, [1, 11, 15]

A complete axiomatization of a behavioural congruence yields a purely syntactic characterization, independent of the actual details of the chosen semantic model for processes and of the definition of the behavioural equivalence, of the semantics of a process algebra. This bridge between syntax and semantics plays an important role in both the practice and the theory of process algebras. From the point of view of practice, these proof systems can be used to perform system verifications in a purely syntactic way using general purpose theorem provers or proof checkers, and form the basis of purpose built axiomatic verification tools. From the theoretical point of view, complete axiomatizations of behavioural equivalences capture the essence of

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different notions of semantics for processes in terms of a basic collection of identities, and this often allows one to compare semantics which may have been defined in very different styles and frameworks. A review of existing complete equational axiomatizations for many of the behavioural semantics in van Glabbeek's spectrum is offered in [26]. The equational axiomatizations offered in that reference are over the language BCCSP, a common fragment of Milner's CCS [21, 22] and Hoare's CSP [19] suitable for describing finite synchronization trees, and characterize the differences between behavioural semantics in terms of a few revealing axioms.

In this paper, we contribute to the study of the equational theory of semantic equivalences over BCCSP by showing that the trace simulation preorder does not have a finite inequational basis over the language BCCSP (Theorem 1). Indeed, no collection of sound inequations of bounded depth is ground-complete with respect to the trace simulation preorder over BCCSP even over a singleton set of actions (Theorem 2). The proof of our main result is proof theoretic. We are sure that Zoltán Ésik would have preferred to see a model-theoretic argument, like those he used with two of the authors of this paper in joint work on the max-plus algebra of the natural numbers and on the equational theory of tropical semirings [2, 3], but we hope that he would have found our result and its proof appealing nonetheless.

The paper is organized as follows. Section 2 presents preliminaries on the syntax and semantics of BCCSP, the behavioural equivalences and preorders we study and inequational logic. Section 3 introduces our main result, whose proof is given in Section 3.1.

2 Preliminaries

Syntax of BCCSP We work with BCCSP [26, 19, 22] over the action set A. This language is a basic process algebra for expressing finite process behaviour. Its syntax consists of closed (process) terms p, q that are constructed from a constant $\mathbf{0}$, a binary operator $\_+\_$ called *alternative composition*, and the unary *prefix* operators $a\_$ with $a \in A$. Open terms t, u can, moreover, contain occurrences of variables from a countably infinite set V (with typical elements x, y, z).

In what follows, for each $n \geq 0$, we use $a^n\mathbf{0}$ to stand for the term $\mathbf{0}$ if $n = 0$, and for $a(a^{n-1}\mathbf{0})$ if $n > 0$.

A (closed) substitution maps variables in V to (closed) terms. For every term t and substitution σ, the term $\sigma(t)$ is obtained by replacing every occurrence of a variable x in t by $\sigma(x)$. Note that $\sigma(t)$ is closed if σ is a closed substitution.

Transition rules Closed BCCSP terms denote finite process behaviours, where $\mathbf{0}$ does not exhibit any behaviour, $p + q$ is the nondeterministic choice between the behaviours of p and q, and ap executes action a to transform into p. This intuition is captured, in the style of Plotkin [25], by the transition rules below, which give rise to a-labelled transitions, with $a \in A$, between closed terms.

$$\frac{}{ax \xrightarrow{a} x} \qquad \frac{x \xrightarrow{a} x'}{x + y \xrightarrow{a} x'} \qquad \frac{y \xrightarrow{a} y'}{x + y \xrightarrow{a} y'}$$

The operational semantics is extended to open terms by assuming that variables do not exhibit any behaviour. We write $t \not\rightarrow$ if there are no action a and term t' such that $t \xrightarrow{a} t'$ holds.

For each $s \in A^*$, the transition relation $\xrightarrow{s}$ is defined thus, where ε denotes the empty string:

- $t \xrightarrow{\varepsilon} t'$ if, and only if, $t = t'$;
- $t \xrightarrow{as} t'$ if, and only if, there is some t'' such that $t \xrightarrow{a} t'' \xrightarrow{s} t'$.

If $t \xrightarrow{s} t'$, then we say that s is a *trace* of t. Such a trace is *complete* if $t' \not\rightarrow$.

For each BCCSP term t, we define

$$T(t) = \{s \mid t \xrightarrow{s} t' \text{ for some } t'\}.$$

The *depth* of a term t, written depth(t), is the length of a longest trace $s \in T(t)$. The *norm* of a term t, written norm(t), is the length of a shortest complete trace $s \in T(t)$. (The notion of norm stems from [6].) For example, the closed term a^2+a^3 has norm two and depth three.

Simulation, bisimulation and trace simulation We define the following three variations on the notion of simulation over closed BCCSP terms.

Definition 1 (Simulations). *A binary relation $\mathcal{R}$ over closed BCCSP terms is:*

- *a* simulation *[20, 24] if $p \mathcal{R} q$ and $p \xrightarrow{a} p'$ imply $q \xrightarrow{a} q'$ for some q' with $p' \mathcal{R} q'$;*
- *a* bisimulation *[22, 24] if it is a simulation whose inverse is also a simulation;*
- *a* trace simulation *if it is a simulation that satisfies the following condition:*

  $p \mathcal{R} q$ *implies* $T(q) = T(p)$.

We write $p \precsim_{TS} q$ if there is a trace simulation $\mathcal{R}$ with $p \mathcal{R} q$, and $p \leftrightarrow q$ if there is a bisimulation $\mathcal{R}$ with $p \mathcal{R} q$. We will refer to $\precsim_{TS}$ as the trace simulation preorder*, and to $\leftrightarrow$ as* bisimilarity*.*

Let $\precsim \in \{\precsim_{TS}, \leftrightarrow\}$. We define $t \precsim u$ if $\sigma(t) \precsim \sigma(u)$ for each closed substitution σ.

It is well known that $\precsim_{TS}$ is a preorder and $\leftrightarrow$ is an equivalence relation. Moreover, both relations are preserved by the operators of the language BCCSP.

Inequational logic An *inequation* (respectively, an *equation*) over the language BCCSP is a formula of the form $t \leq u$ (respectively, $t = u$), where t and u are BCCSP terms. An *(in)equational axiom system* is a collection of (in)equations over the language BCCSP. An equation $t = u$ is derivable from an equational axiom system E if it can be proven from the axioms in E using the rules of equational logic (viz. reflexivity, symmetry, transitivity, substitution and closure under BCCSP contexts).

$$t = t \quad \frac{t = u}{u = t} \quad \frac{t = u \quad u = v}{t = v} \quad \frac{t = u}{\sigma(t) = \sigma(u)} \quad \frac{t = u}{at = au} \quad \frac{t = u \quad t' = u'}{t + t' = u + u'}$$

For the derivation of an inequation $t \leq u$ from an inequational axiom system E, the rule for symmetry is omitted.

It is well known that, without loss of generality, one may assume that substitutions happen first in (in)equational proofs, i.e., that the fourth rule may only be used when its premise is one of the (in)equations in E. Moreover, by postulating that for each equation in E also its symmetric counterpart is present in E, one may assume that applications of symmetry happen first in equational proofs, i.e., that the second rule is never used in equational proofs. (See, e.g., [12, page 497] for a thorough discussion of this 'normalized equational proofs'.) In the remainder of this paper, we shall always tacitly assume that equational axiom systems are closed with respect to symmetry. Note that, with this assumption, there is no difference between the rules of inference of equational and inequational logic. In what follows, we shall consider an equation $t = u$ as a shorthand for the pair of inequations $t \leq u$ and $u \leq t$.

The depth of $t \leq u$ and $t = u$ is the maximum of the depths of t and u. The depth of a collection of (in)equations is the supremum of the depths of its elements.

An inequation $t \leq u$ is *sound* with respect to $\precsim_{TS}$ if $t \precsim_{TS} u$ holds. For example, as our readers can readily check, the inequation

$$ax \leq ax + x \qquad (1)$$

is sound with respect to $\precsim_{TS}$ if $A = \{a\}$ and is unsound otherwise.

An (in)equational axiom system E is sound with respect to $\precsim_{TS}$ if so is each (in)equation in E. It is *complete* if each valid inequation $t \precsim_{TS} u$ can be derived from E, and it is *ground complete* if each valid inequation $t \precsim_{TS} u$ relating *closed terms* can be derived from E. A set of complete and sound (in)equations is sometimes referred to as an *(in)equational basis.*

The core axioms A1–A4 for BCCSP given below are classic and stem from [18]. They are complete [23], and sound and ground complete [18, 22], over BCCSP (over any nonempty set of actions) modulo bisimulation equivalence [22, 24], which is the finest semantics in van Glabbeek's spectrum [26].

A1	$x + y$	$\approx$	$y + x$
A2	$(x + y) + z$	$\approx$	$x + (y + z)$
A3	$x + x$	$\approx$	x
A4	$x + \mathbf{0}$	$\approx$	x

In what follows, for notational convenience, we consider terms up to the least congruence generated by axioms A1–A4, that is, up to bisimulation equivalence.

3 The negative result

Our aim in what follows is to show the following theorem.

Theorem 1. *The (in)equational theory of $\precsim_{TS}$ over BCCSP does not have a finite inequational basis. In particular, no finite set of sound inequations over BCCSP modulo $\precsim_{TS}$ can prove all of the sound inequations in the family*

$$a^{2m} \leq a^{2m} + a^m \qquad (m \geq 0).$$

In what follows, we shall present a proof of the above result, which has proof has a 'proof-theoretic' flavour.

Remark 1. The family of inequations in the statement of Theorem 1 was used in [5, 4] to prove that the 2-nested simulation preorder from [17] does not afford a finite ground-complete inequational axiomatization over BCCSP.

3.1 A proof-theoretic argument for Theorem 1

Our proof of Theorem 1 is based on obtaining that result as a corollary of the following one.

Theorem 2. *Let E be a collection of inequations whose elements are sound modulo $\precsim_{TS}$ and have depth smaller than m. Suppose furthermore that the closed inequation $p \leq q$ is derivable from E, that $q \precsim_{TS} a^{2m} + a^m$ and* $\mathrm{norm}(p) = 2m$*. Then* $\mathrm{norm}(q) = 2m$*.*

Having shown the above result, Theorem 1 can be proved as follows. Let E be a finite inequational axiom system that is sound modulo $\precsim_{TS}$. Pick m larger than the depth of E. (Such an m exists since E is finite.) Then, by Theorem 2, E cannot prove the valid inequation

$$a^{2m} \leq a^{2m} + a^m,$$

and is therefore incomplete. Indeed, a^{2m} has norm $2m$, but $a^{2m} + a^m$ has norm m.

In the remainder of this section, we shall present a proof of Theorem 2. In order to show that result, we shall first prove that the property mentioned in that statement holds true for instantiations of sound inequations whose depth is smaller than m. Next we use this fact to argue that the stated property is preserved by arbitrary inequational derivations from a collection of inequations whose elements have depth smaller than m and are sound modulo $\precsim_{TS}$.

Definition 2. *We say that a term t has an occurrence of variable x reachable via a sequence of actions s if there is some term t' such that $t \xrightarrow{s} x + t'$.*

For example, $ax + a\mathbf{0}$ has an occurrence of x reachable via a because $ax + a\mathbf{0} \xrightarrow{a} x$ and $x = x + \mathbf{0}$.

Lemma 1. *Assume that $t \precsim_{TS} u$ and that u has an occurrence of variable x reachable via a sequence of actions s. Then t also has an occurrence of variable x reachable via some sequence of actions s'.*

Proof. Assume that $t \precsim_{TS} u$ and that u has an occurrence of variable x reachable via a sequence of actions s. Let m be larger than the depth of t. Consider the closed substitution σ mapping x to a^m and every other variable to $\mathbf{0}$. Since u has an occurrence of variable x reachable via s, it is easy to see that $\sigma(u) \xrightarrow{sa^m} \mathbf{0}$. As $\sigma(t) \precsim_{TS} \sigma(u)$ because $t \precsim_{TS} u$ by assumption, it must be the case that $\sigma(t) \xrightarrow{sa^m} p$ for some p. As the depth of t is smaller than m, the substitution σ maps all variables different from x to $\mathbf{0}$ and $\sigma(u) \xrightarrow{sa^m} p$, it follows that $t \xrightarrow{s'} x + t'$ for some t', which was to be shown. □

Remark 2. Note that, in general, the traces s and s' mentioned in the statement of the above lemma need not be equal. For instance, as we observed previously, the inequation

$$ax \leq ax + x$$

is sound with respect to $\precsim_{TS}$ if $A = \{a\}$ and the term $ax + x$ has an occurrence of variable x reachable via the sequence of actions ε. However, the only occurrence of x in the term ax is reachable via the sequence of actions a.

The following lemma is the first stepping stone towards the proof of Theorem 2. It establishes that the property mentioned in that statement holds true for instantiations of sound inequations whose depth is smaller than m.

Lemma 2. *Suppose that $t \precsim_{TS} u$ and that m is larger than the depth of u. Let σ be a closed substitution. Suppose, furthermore, that $\sigma(u) \precsim_{TS} a^{2m} + a^m$ and $\mathrm{norm}(\sigma(t)) = 2m$. Then $\mathrm{norm}(\sigma(u)) = 2m$.*

Proof. The assumption that $\sigma(u) \precsim_{TS} a^{2m} + a^m$ yields that $\mathrm{norm}(\sigma(u)) = 2m$ or $\mathrm{norm}(\sigma(u)) = m$. Assume, towards a contradiction, that $\mathrm{norm}(\sigma(u)) = m$. Then, since $\mathrm{depth}(u) < m$, there are some $i < m$ and some variable x such that u has an occurrence of variable x reachable via a^i and $\sigma(x) \xrightarrow{a^{m-i}} \mathbf{0}$. Since $t \precsim_{TS} u$ and $\mathrm{depth}(t) < m$ too (because $t \precsim_{TS} u$ clearly implies that $\mathrm{depth}(t) = \mathrm{depth}(u)$ and $\mathrm{depth}(u) < m$ by our assumption), there is some $j < m$ such that t has an occurrence of variable x reachable via a^j. But then $\sigma(t)$ has a trace of length $j + (m - i) < 2m$ leading to $\mathbf{0}$. This contradicts the assumption that $\mathrm{norm}(\sigma(t)) = 2m$. Therefore $\mathrm{norm}(\sigma(u)) = 2m$, as claimed. □

We will now argue that the property stated in Theorem 2 is preserved by arbitrary inequational derivations from a collection of inequations whose elements are sound modulo $\precsim_{TS}$ and have depth smaller than m. The following lemma will allow us to handle closure under action prefixing in that proof.

Lemma 3. *Assume that $aq \precsim_{TS} a^{2m} + a^m$. Then $\mathrm{norm}(aq) = 2m$.*

Proof. By our assumptions, it follows that $m \geq 1$, $\text{depth}(aq) = 2m$ and that $\text{norm}(aq) = 2m$ or $\text{norm}(aq) = m$.

Assume, towards a contradiction, that $\text{norm}(aq) = m$. Then q has depth $2m-1$ and norm $m-1$. Since $aq \precsim_{TS} a^{2m} + a^m$ and $\text{depth}(q) = 2m-1$, it must be the case that $q \precsim_{TS} a^{2m-1}$. But this is impossible since q can terminate in $m-1$ steps and a^{2m-1} cannot. Therefore aq has norm $2m$, as claimed. □

We now have all the necessary ingredients to complete our proof of Theorem 2, and therefore of Theorem 1.

Proof. (of Theorem 2) Assume that E is a collection of inequations whose elements are sound modulo $\precsim_{TS}$ and have depth smaller than m. Suppose furthermore that

- the inequation $p \leq q$ is derivable from E,
- $q \precsim_{TS} a^{2m} + a^m$, and
- $\text{norm}(p) = 2m$.

(Observe that m is positive because it is larger than the depth of E.) We shall prove that $\text{norm}(q) = 2m$ by induction on a closed derivation of $p \leq q$ from E. We proceed by examining the last rule used in the proof of $p \leq q$ from E. The case of reflexivity is trivial and that of transitivity follows by applying the inductive hypothesis twice. If $p \leq q$ is proved by instantiating an inequation in E, then the claim follows by Lemma 2. We are therefore left with the congruence rules, which we examine separately below.

- Suppose that E proves $p \leq q$ because $p = ap'$, $u = aq'$ and E proves $p' \leq q'$ by a shorter inference. By the soundness of E and the proviso of the theorem, we have that
$$p = ap' \precsim_{TS} u = aq' \precsim_{TS} a^{2m} + a^m$$
and $\text{norm}(p) = 2m$. Lemma 3 now yields $\text{norm}(q) = 2m$, as required.
- Suppose that E proves $p \leq q$ because $p = p_1 + p_2$, $q = p_1 + p_2$ and E proves $p_i \leq q_i$, $1 \leq i \leq 2$, by shorter inferences. Since p has norm $2m$ and m is positive, we may assume, without loss of generality, that p_1 has norm $2m$. Moreover, the depth of p_1 is also $2m$, since
$$p = p_1 + p_2 \precsim_{TS} q_1 + q_2 = q \precsim_{TS} a^{2m} + a^m.$$
Therefore q_1 has depth $2m$ because E is sound. Since $q_1 + q_2 \precsim_{TS} a^{2m} + a^m$, for each q_1' such that $q_1 \xrightarrow{a} q_1'$ we have that $q_1' \precsim_{TS} a^{2m-1}$ or $q_1' \precsim_{TS} a^{m-1}$. As q_1 has positive depth, this means that $q_1 \precsim_{TS} a^{2m} + a^m$. We may therefore apply the induction hypothesis to obtain that $\text{norm}(q_1) = 2m$. If p_2 is $\mathbf{0}$ then we are done since, in that case, q_2 is also $\mathbf{0}$ by the soundness of E. If p_2 is not $\mathbf{0}$, then its norm is also $2m$, because p has norm and depth equal to $2m$. But then, reasoning as above, we may infer that $\text{norm}(q_2) = 2m$. Since $q = q_1 + q_2$ and $\text{norm}(q_1) = \text{norm}(q_2) = 2m$, we have that $\text{norm}(q) = 2m$, which was to be shown.

This completes the proof. □

Dedication Luca Aceto and Anna Ingólfsdóttir dedicate this paper to the memory of their collaborator and friend Zoltán Ésik, from whom they have learned much and with whom they have shared many pleasant days. They will miss him.

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